8.1 Inferential Statistics

μ - the true population mean
p - the true population proportion

estimate μ with $\bar{x}$ (estimators)
estimate p with $\hat{p}$

$\bar{x}$ and $\hat{p}$ are point estimates

Interval Estimate -
- margin of error - add & subtract from $\bar{x}$ or $\hat{p}$ to get an interval.

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The margin error depends on

(1) The standard deviation $\sigma_{\bar{x}}$ of $\bar{x}$

(2) The level of confidence to be attached.

Interval

point estimate $\pm$ margin of error

confidence interval

Confidence Level is denoted

$(1-\alpha)100\%$

$1-\alpha$ confidence coefficient

α is called the significance level.

90%, 95%, 99%

confidence levels

$(1-\alpha)100 = 95$

$$1-\alpha = .95$$
$$\underline{-.95+\alpha \quad -.95+\alpha}$$
$$1-.95 = \alpha$$
$$.05$$

99%

$\alpha = 1-.99$

$= .01$

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8.3

σ is known

Estimate Population mean

CASE I CASE II

Can use

$\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ given $\frac{n}{N} \leq .05$

The confidence Interval for μ

the $(1-\alpha)100\%$ confindence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}}$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

margin of error $z \, \sigma_{\bar{x}}$

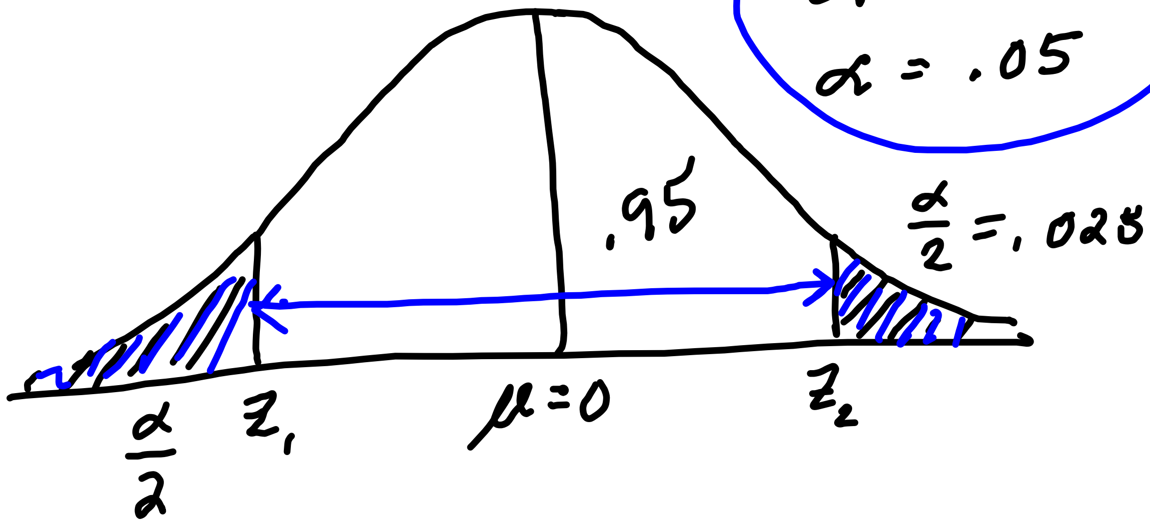
$z \rightarrow$ # of s.d.

$\sigma_{\bar{x}} \rightarrow$ s.d

$$E = z\,\sigma_{\bar{x}}$$

what is z for a 95% ~~confidence~~ level?

$$\alpha = 1 - .95$$
$$\alpha = .05$$



$$z = -\text{invNorm}(.025, 0, 1)$$
$$z \approx 1.95996$$

round at the end.

Find Z for a 99% confidence level

$\alpha = 1 - .99 = .01$

$\frac{\alpha}{2} = .005$

$\alpha = .01$

$\frac{\alpha}{2} = \frac{.01}{2} = .005$

$Z = \text{invNorm}(.005)$

$Z \approx 2.5758$

EX 8-1 25 textbooks

pg 343 $\bar{x} = \$90.50$ $\sigma = \$7.50$

$n = 25$, $\bar{x} = 90.50$, $\sigma_{\bar{x}} = \frac{7.50}{\sqrt{25}} = 1.50$

a) Find the point estimate of μ

$\bar{x} = \$90.50$

b) Construct a 90% confidence interval

$\alpha = 1 - .90 = .10$

$\frac{\alpha}{2} = .05$

$z = \text{invNorm}(.05)$
$= 1.6449$

$E = 1.6449(1.50) \approx 2.4674$

2.47

always round up.

$90.50 - 2.47 = 88.03$

$90.50 + 2.47 = 92.97$

answer $(88.03, 92.97)$ 90% confidence interval.

EX 8-2 $\bar{x} = \$7868$ in 2004

$n = 900$ $\sigma = 2070$ $\sigma_{\bar{x}} = \frac{2070}{\sqrt{900}} = 69$

Make a 99% confidence interval for μ

$\alpha = .01$ $\frac{\alpha}{2} = .005$

$z = \text{invNorm}(.005)$

≈ 2.5758

$E = z \cdot \sigma_{\bar{x}} = 2.5758(69)$

margin of error ≈ 177.7302

≈ 177.74 round up.

$7868 - 177.74 = 7690.26$

$7868 + 177.74 = 8045.74$

99% confidence level $(7690.26, 8045.74)$

The width of the confidence interval depends on

1. confidence level (Z)
2. The sample size (n)

To make the confidence interval smaller we <u>can</u>

1. Lower the confidence level
2. Increase the sample size

8.14 population $\sigma = 14.8$

population is normally distributed

sample = 25

$\bar{x} = 143.72$, $\sigma_{\bar{x}} = \frac{14.8}{\sqrt{25}} = 2.96$

a) Make a 99% confidence interval

$\alpha = .01$

$z = \text{invNorm}(.005)$ $\frac{\alpha}{2} = .005$

$z \approx 2.5758$

$E = 2.5758(2.96)$

≈ 7.6245

≈ 7.63

$143.72 - 7.63 = 136.09$

$143.72 + 7.63 = 151.35$

$(136.09, 151.35)$ 99% confidence interval

$n = 25$

b) Construct 95% confidence interval

$z = \text{invNorm}(.025)$ $\alpha = .05$

$z \approx 1.959964$ $\frac{\alpha}{2} = .025$

≈ 1.9600

margin of error $= 1.9600(2.96)$

≈ 5.80149

≈ 5.81

$143.72 - 5.81 = 137.91$

$143.72 + 5.81 = 149.53$

$(137.91, 149.53)$ 95% confidence level

If we increase the sample size
n = 40 observations
99% confidence level

$$\sigma_{\bar{x}} = \frac{14.8}{\sqrt{40}} \approx 2.3400854$$

$$E = 2.575829(2.340085)$$
$$\approx 6.02766$$
$$\approx 6.03$$

$$143.72 - 6.03 = 137.69$$
$$143.72 + 6.03 = 149.75$$

(137.69, 149.75) 99% confidence interval

Determining sample size for an Estimate of the Mean

$$E = z\,\sigma_{\bar{x}}$$

$$E = z\,\frac{\sigma}{\sqrt{n}}$$

If we know what margin of error we want & the confidence level, we can determine the sample needed.

$$\sqrt{n}\,E = z\,\frac{\sigma}{\cancel{\sqrt{n}}}\left(\cancel{\sqrt{n}}\right)$$

$$\frac{E\sqrt{n}}{E} = \frac{z\,\sigma}{E}$$

$$\left(\sqrt{n}\right)^2 = \left(\frac{z\,\sigma}{E}\right)^2$$

$$n = \frac{z^2\,\sigma^2}{E^2}$$

EX 8-3 debt of all this year's graduates

$\sigma = \$11,800$

How large a sample should be selected so that a 99% confidence level has a margin of error of \$800 of the population mean?

$E = 800$ $\quad z = 2.57583$

$\sigma = 11800$

$$n = \frac{z^2 \sigma^2}{E^2}$$

$$n = \frac{(2.57583)^2 (11800)^2}{(800)^2}$$

$$\approx 1443.58$$

$\therefore$ n should equal 1444 people

Check

$E = z\,\sigma_{\bar{x}}$

$\sigma_{\bar{x}} = \dfrac{11806}{\sqrt{1444}}$

≈ 310.53

$E \doteq 2.5759(310.53)$

≈ 799.894

8.12

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$n = 81 \quad \bar{x} = 48.25 \quad \sigma = 4.8$

a) What is the pt estimate of μ?

48.25

b) 95% confidence interval for μ.

$\alpha = 1 - .95 = .05$

$\frac{\alpha}{2} = .025$

$\sigma_{\bar{x}} = \frac{4.8}{9} \approx .5333$

$z = 1.9600$

$E = 1.9600(.5333)$

≈ 1.0453

≈ 1.05

$48.25 - 1.05 \approx 47.20$

$48.25 + 1.05 \approx 49.30$

$(47.20, 49.30)$ 95% confidence interval.

c) $E \approx 1.05$

8.20 $\sigma = 14.50$

a) 98% confidence interval for μ to have $E = 5.50$

$\alpha = .02$

$\frac{\alpha}{2} = .01$

$z = \text{invNorm}(.01)$

≈ 2.3263

$E = 5.50$

$$n = \frac{z^2 \sigma^2}{E^2}$$

$$n = \frac{(2.3263)^2 (14.50)^2}{(5.50)^2}$$

≈ 37.6134

$n \approx 38$

b) 95% $E = 4.25$ $\sigma = 14.50$

$\alpha = .05$ $z = 1.9600$

$\frac{\alpha}{2} = .025$

$$n = \frac{(1.9600)^2 (14.50)^2}{(4.25)^2}$$

$= 44.7$

$n = 45$