

No class Friday 4/27/12.

8.1

Roots and Radicals

Common Roots

$$5^2 = 25$$

↑

5 is a square root of 25

$$\sqrt{25}$$

√

radical

25 is the radicand

$\sqrt{25}$ is a radical expression

$\sqrt{64}$ principle are positive square root

$$8 \cdot 8 = 64$$

$$(-8)(-8) = 64$$

$$\sqrt{64} = 8$$

$$-\sqrt{64} = -8$$

If a number is not a perfect square
say 11

$$\sqrt{11}$$

irrational number.

One number that has one square root

$$\sqrt{0} = 0$$

$$\sqrt{1} = 1$$

$$-\sqrt{1} = -1$$

$\sqrt{-4}$ is not real

$$2 \cdot 2 = 4$$

$$(-2)(-2) = 4$$

$$-\sqrt{4} = -2$$

$$\sqrt{4} = 2$$

$$\pm\sqrt{4} = \pm 2$$

$$\sqrt[n]{}$$

$$\sqrt[3]{8} = 2$$

$$\sqrt[4]{16} = 2$$

$$\sqrt[4]{-16} \text{ not real}$$

$$\sqrt[3]{-27} = -3$$

$$\sqrt[5]{-32} = -2$$

$$\sqrt[3]{-125} = -5$$

$$\sqrt[4]{-81} \text{ not real}$$

$$-\sqrt[4]{81} = -3$$

We will assume all our variables under the radical signs are positive

Ex

$$\sqrt{x^2} = x$$

$$\sqrt{9y^2} = 3y$$

$$\sqrt[3]{x^9} = x^3$$

$$\sqrt{x^2 y^4} = xy^2$$

$$\sqrt[3]{8a^6} = 2a^2$$

Divide the index
into the power

Simplify

$$\frac{3 + \sqrt{25}}{4}$$

$$\frac{3 + 5}{4}$$

$$\frac{8}{4}$$

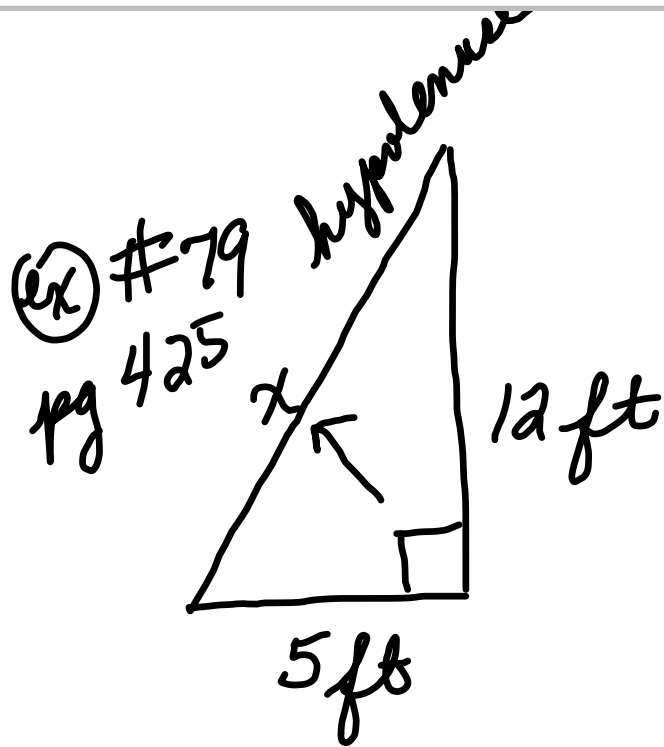
$$2$$

$$\frac{3 - \sqrt{25}}{4}$$

$$\frac{3 - 5}{4}$$

$$-\frac{2}{4}$$

$$-\frac{1}{2}$$



right triangle

Pythagorean
Theorem

$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = x^2$$

$$25 + 144 = x^2$$

$$\sqrt{169} = \sqrt{x^2}$$

$$13 = x$$

x is 13 feet

8.2

Properties of Radicals

$$\sqrt{25 \cdot 16} = \sqrt{400} = 20$$

$$\begin{aligned}\sqrt{400} &= \sqrt{25} \cdot \sqrt{16} \\ &= 5 \cdot 4 \\ &= 20\end{aligned}$$

If x and y represent non-neg. numbers.

It is always true

$$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

$$\sqrt{x} \sqrt{y} = \sqrt{xy}$$

$$\begin{aligned}
 \textcircled{\text{ex}} \quad \sqrt{18} &= \sqrt{9 \cdot 2} \\
 &= \sqrt{9} \sqrt{2} \\
 &= 3\sqrt{2} \text{ simplest form}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{\text{ex}} \quad 3a \sqrt[3]{27a^4} \\
 3a \sqrt[3]{27a^3a} \\
 3a(3a) \sqrt[3]{a} \\
 9a^2 \sqrt[3]{a}
 \end{aligned}$$

ex)

$$\sqrt{\frac{64}{16}} = \sqrt{4} = 2$$

$$\frac{\sqrt{64}}{\sqrt{16}} = \frac{8}{4} = 2$$

$y \neq 0$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

$$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$$

Q4 Simplify

$$(30) \sqrt{\frac{81}{64}} = \frac{\sqrt{81}}{\sqrt{64}} = \frac{9}{8}$$

$$(34) \sqrt[3]{\frac{64}{27}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3}$$

$$(40) \sqrt{\frac{64a^2b^2}{16}} = \frac{\sqrt{64} \sqrt{a^2} \sqrt{b^2}}{\sqrt{16}} \\ = \frac{8ab}{4} \\ = 2ab$$

8.3

Simplified Form for Radicals

Definition - A radical is in simplest form if

- ① There are no perfect powers that match the index
- ② There are no fractions under the radical
- ③ There are no radicals in the denominator!

Property $\checkmark$

$$\sqrt{a} \sqrt{a} = \sqrt{a^2} = a$$
$$= (\sqrt{a})^2$$

ex) $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

Rationalizing the
denominator

$$\textcircled{ex} \quad \frac{4\sqrt{12}}{8\sqrt{3}} = \frac{\overset{1}{4}\overset{1}{\sqrt{4}}\sqrt{3}}{\underset{2}{8}\underset{1}{\sqrt{3}}} = \frac{2\sqrt{3}}{2\sqrt{3}} = 1$$

$$\begin{aligned} \textcircled{ex} \quad \frac{2\sqrt{45x^4}}{\sqrt{5x}} &= 2\sqrt{\frac{45x^4}{5x}} = 2\sqrt{9x^3} \\ &= 2\sqrt{9x^2x} \\ &= 2(3)x\sqrt{x} \\ &= 6x\sqrt{x} \end{aligned}$$

$$\textcircled{24} \quad \sqrt[3]{\frac{7}{9}} = \frac{\sqrt[3]{7}}{\sqrt[3]{9} \sqrt[3]{3}} = \frac{\sqrt[3]{21}}{3}$$

$$\rightarrow \sqrt[3]{27}$$

rationalizing the denominator

8.4

Addition & Subtraction of Radical Expressions

$$\begin{array}{r} 7\sqrt{2} + 2\sqrt{2} + \sqrt{3} \\ 9\sqrt{2} + \sqrt{3} \end{array}$$

(ex)
$$\begin{array}{r} 7\sqrt{2} + 2\sqrt{2} + \sqrt{3} \\ 9\sqrt{2} + \sqrt{3} \end{array}$$

(ex)
$$11\sqrt{11} + \sqrt{11} = 12\sqrt{11}$$

ex)

$$\sqrt{20} + 3\sqrt{5}$$

4

$$\sqrt{4 \cdot 5} + 3\sqrt{5}$$

$$2\sqrt{5} + 3\sqrt{5}$$

$$5\sqrt{5}$$

ex) $\frac{1}{2}\sqrt{24} + \frac{1}{5}\sqrt{150}$

$$\frac{1}{2}\sqrt{4 \cdot 6} + \frac{1}{5}\sqrt{25 \cdot 6}$$

$$\frac{1}{2}(2)\sqrt{6} + \frac{1}{5}(5)\sqrt{6}$$

$$\sqrt{6} + \sqrt{6}$$

$$2\sqrt{6}$$

ex)

$$5\sqrt{50} + 8\sqrt{12} - \sqrt{32}$$

$$5\sqrt{25 \cdot 2} + 8\sqrt{4 \cdot 3} - \sqrt{16 \cdot 2}$$

$$5(5)\sqrt{2} + 8(2)\sqrt{3} - 4\sqrt{2}$$

$$25\sqrt{2} + 16\sqrt{3} - 4\sqrt{2}$$

$$21\sqrt{2} + 16\sqrt{3} \text{ done}$$

Ex

6

$$9\sqrt{24x^3y^2} - 5x\sqrt{xy^2}$$

$$9\sqrt{\underline{4} \cdot \underline{6} x^2 \underline{y} y^2} - 5x\sqrt{xy^2}$$

$$9(2xy\sqrt{6x}) - 5xy\sqrt{x}$$

$$18xy\sqrt{6x} - 5xy\sqrt{x} \quad \text{done}$$

can't add

8.5

$$\sqrt{a} \sqrt{b} = \sqrt{ab}$$

$$\textcircled{\text{ex}} \sqrt{6} \cdot \sqrt{3} = \sqrt{18} = \sqrt{9 \cdot 2} \\ = 3\sqrt{2}$$

$$\textcircled{\text{ex}} (7\sqrt{6})(3\sqrt{2}) = 21\sqrt{12} \\ = 21\sqrt{4 \cdot 3} \\ = 42\sqrt{3}$$

$$\textcircled{\text{ex}} \sqrt{7}(\sqrt{14} - \sqrt{7}) \\ \sqrt{7 \cdot 7 \cdot 2} - \sqrt{49} \\ 7\sqrt{2} - 7$$

$$\textcircled{\text{ex}} \quad (\sqrt{5} - 4)^2 = 5 - 4\sqrt{5}(2) + 16 \quad \leftarrow$$

$$= 21 - 8\sqrt{5}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$\begin{array}{r} \sqrt{5} - 4 \\ \sqrt{5} - 4 \\ \hline 5 - 4\sqrt{5} \\ - 4\sqrt{5} + 16 \\ \hline 5 - 8\sqrt{5} + 16 \end{array}$$

$$\textcircled{\text{ex}} (\sqrt{7} + 4)(\sqrt{7} - 4) = 7 - 16$$

conjugates = -9



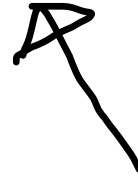
$$\sqrt{49} = 7$$

$$\textcircled{\text{ex}} (\sqrt{x} + 7)(\sqrt{x} - 7) = x - 49$$

$$\textcircled{\text{ex}} (3\sqrt{a} - 2\sqrt{b})(3\sqrt{a} + 2\sqrt{b}) = 9a - 4b$$

$$(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = 3 - 2 = 1$$

$$\frac{1}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$



$$\left(\frac{\sqrt{2}}{\sqrt{6} + \sqrt{3}} \right) \left(\frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} - \sqrt{3}} \right) = \frac{\sqrt{12} - \sqrt{6}}{6 - 3} = \frac{2\sqrt{3} - \sqrt{6}}{3}$$

multiply by the conjugate
of the denominator

$$\begin{aligned} \left(\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \right) \left(\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \right) &= \frac{3 + 2\sqrt{6} + 2}{3 - 2} \\ &= \frac{5 + 2\sqrt{6}}{1} \\ &= 5 + 2\sqrt{6} \end{aligned}$$