#18
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10% alcohol
5% alcohol
40 liters of 8% alcohol

let x = the amount of 10% alcohol
y = the " " 5% alcohol

	amt	conc.	amt of pure alcohol
10%	x	.10	$.10x$
5%	y	.05	$.05y$
mix	40	.08	$.08(40)$

$$100\left[.10x + .05y = .08(40)\right]$$

$$x + y = 40$$
$$-x \quad -x$$
$$y = 40 - x$$

$$\left[.10x + .05(40 - x) = .08(40)\right]100$$

$$10x + 5(40 - x) = 8(40)$$
$$10x + 200 - 5x = 320$$
$$5x + 200 = 320$$
$$-200 \quad -200$$
$$\frac{5x}{5} = \frac{120}{5}$$
$$x = 24$$
$$40 - 24 = 16$$

multiplication

$100\left(.05(40 - x)\right)$

$.08(40)(100)$
$100(.08)(40)$

We would need 24 liters of the 10% solution and 16 liters of the 5% solution.

Chapter 5

5.1 Exponents

5^2 → the exponent

5^2 ← the base

$\square^{\square}$

an exponential expression

$-a^4$ a is the base

$(-a)^4$ $-a$ is the base

$-2^4 = -16$

$(-2)^4 = 16$

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

$2^3 = 2 \cdot 2 \cdot 2$

$2^4 \cdot 2^3 = (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 2^7$

$2^4 \cdot 2^3 = 2^{4+3} = 2^7 = 128$

$x^2 \cdot x^7 = x^9$

$y^1 \cdot y^7 = y^8$

PROPERTY I
Product Rule
For any integers
n and m
$a^m \cdot a^n = a^{m+n}$

Must be the same.

x^2

$$2^3 \cdot 2^2 = 2^5 = 32$$

$$\underbrace{2^3 + 2^2} = 8 + 4 = 12$$

$$3x^2(2x^7)$$

$$3 \cdot 2 \cdot x^2 \cdot x^7$$

$$6x^9$$

$$3 \cdot x^2$$

$$2 \cdot x^7$$

What if we take a power to a power?

$(6^2)^3 = 6^2 \cdot 6^2 \cdot 6^2$

$= 6^6$

$6^{2 \cdot 3} = 6^6$

$(y^3)^5 = y^{15}$

You multiply the exponents

$(a^m)^n = a^{m \cdot n}$

$(8^4)^2 = 8^8$

$(6 \cdot 2)^3 = (6 \cdot 2)(6 \cdot 2)(6 \cdot 2)$

$= 6 \cdot 2 \cdot 6 \cdot 2 \cdot 6 \cdot 2$

$= 6 \cdot 6 \cdot 6 \cdot 2 \cdot 2 \cdot 2$

$(6 \cdot 2)^3 = 6^3 \cdot 2^3$

PROPERTY III $(ab)^m = a^m b^m$

ex $(2x)^5 = 2^5 x^5$

$= 32x^5$

ex Simplify $(3xy)^2$

$3^2 x^2 y^2$

$9x^2y^2$

Ex. $3(2m^2p^3)^4 = 3(2^4 m^8 p^{12})$

$= 3(16 m^8 p^{12})$

$= 48 m^8 p^{12}$

Since $\frac{a}{b}$ can be written $a \cdot (\frac{1}{b})$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad b \neq 0$$

$$\left(\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)\left(\frac{5}{2}\right) = \frac{5 \cdot 5}{2 \cdot 2} = \frac{25}{4}$$

Ex. Simplify $\dfrac{(5x)^3(5x)^4}{(5x)^7}$

$5^7 x^7$

$3625x^7$

(ex) $(2x^2y^3)^4(3xy^2)^3$

$2^4x^8y^{12} \cdot 3^3x^3y^6$

$2^4 3^3 x^8 x^3 y^{12} y^6$

$16(27)x^{11}y^{18}$

$432x^{11}y^{18}$

$$\begin{array}{r} 27 \\ 16 \\ \hline 162 \\ 27 \\ \hline 432 \end{array}$$

③ Green Ron has 14 coins
total value $2.30
nickels and quarters

let x = the number of nickels
y = the # of quarters

	amt	value	total worth
nickels	x	.05	$.05x$
quarters	y	.25	$.25y$
total	14		2.30

$-5(x + y = 14)$

$(.05x + .25y = 2.30)\,100$

$5x + 25y = 230$

$-5x - 5y = -70$
$5x + 25y = 230$

$\frac{20y}{20} = \frac{160}{20}$

$y = 8$

$14 - 8 = 6$

Ron has 8 quarters and 6 nickels.

① Yellow

$$\begin{aligned} -4x - 2y &= 3 \\ 2(2x + y &= 1) \end{aligned} \rightarrow$$

$$\begin{aligned} -4x - 2y &= 3 \\ 4x + 2y &= 2 \end{aligned}$$

$$0 + 0 = 5$$

$$0 = 5$$

false

∴ no solutions

② $$\begin{aligned} 4x - 2y &= 8 \\ 2(3x + y &= -19) \end{aligned} \rightarrow$$

$$\begin{aligned} 4x - 2y &= 8 \\ 6x + 2y &= -38 \end{aligned}$$

$$\frac{10x}{10} = \frac{-30}{10}$$

$$x = -3$$

$$3(-3) + y = -19$$

$$-9 + y = -19$$

$$+9 \qquad +9$$

$$y = -10$$

$$(-3, -10)$$

③ 15 coins 3.15 dimes quarters

① let x = the # of dimes
y = the # of quarters

	#	worth	total amt
dimes	x	.10	$.10x$
quarters	y	.25	$.25y$
total	15		3.15

② $x + y = 15$

$100(.10x + .25y = 3.15)$

$x + y = 15$
$-x \quad -x$
$y = 15 - x$

$10x + 25(15 - x) = 315$

$10x + 375 - 25x = 315$

$-15x + 375 = 315$

$-375 \quad -375$

$\frac{-15x}{-15} = \frac{-60}{-15}$

④

③ $x = 4$

$15 - 4 = 11$

So, he has 4 dimes and 11 quarters.

2

Given $x - 4y = 2$

$-3x + 12y = -8$

$x = 2 + 4y$

$-3(2 + 4y) + 12y = -8$

$-6 - 12y + 12y = -8$

$-6 = -8$ FALSE

no solution

(1)

$$-3x + 4y = 1$$
$$-4x + y = -3$$

$y = -3 + 4x$

$$-3x + 4(-3+4x) = 1$$
$$-3x - 12 + 16x = 1$$
$$13x - 12 = 1$$
$$+12 \quad +12$$
$$13x = 13$$
$$\frac{13x}{13} = \frac{13}{13}$$
$$x = 1$$

$y = -3 + 4(1)$
$y = 1$

$(1, 1)$

$$\begin{array}{r} -3x + 4y = 1 \\ -4(-4x + y = -3) \end{array} \rightarrow$$

$$\begin{array}{r} -3x + 4y = 1 \\ 16x - 4y = 12 \\ \hline 13x = 13 \end{array}$$

$$\frac{13x}{13} = \frac{13}{13}$$

$x = 1$ $(1,1)$

$$-4(1) + y = -3$$

$$\begin{array}{r} -4 + y = -3 \\ +4 \quad\quad +4 \\ \hline y = 1 \end{array}$$

$$-3(1) + 4y = 1$$

$$\begin{array}{r} -3 + 4y = 1 \\ +3 \quad\quad +3 \\ \hline 4y = 4 \end{array}$$

$$y = 1$$