Section 1.3 Adding Whole Numbers

An arithmetic operation is a process that combines quantities to create a new quantity. The four primary arithmetic operations are addition, subtraction, multiplication, and division. Addition is the arithmetic operation denoted by the + symbol that calculates the total formed by combining two quantities.

> The **sum** of the numbers *a* and *b* written $a + b$ is the total obtained from adding the numbers *a* and *b* which are called the **addends**.

There are various key words that can be used to indicate addition. Below the sum with addends 2 and 3 is described using some of the key words.

The **sum** of 2 and 3	2 **plus** 3
3 **more than** 2	2 **increased** by 3
3 **greater than** 2	The **total** of 2 and 3
\$3 **deposited** into an account with a balance of \$2	

Example 1 Identify the addends and the sum in the problem $7 + 2 = 9$

7	+	**2**	=	**9**
addend		addend		sum

7 and 2 are the addends and 9 is the sum.

Example 2 Translate the following into numerical expressions.

The sum of 4 and 2	4 + 2	6 more than 5	5 + 6
9 increased by 6	9 + 6	total of 7 and 15	7 + 15
5 plus 11	5 + 11	8 greater than 12	12 + 8
\$23 deposited into an account with a balance of \$475			475 + 23

To add whole numbers, the two numbers are written with their place values lined up and the digits with same place values are added. Start at the right by adding the digits in the ones place, then the tens place, the hundreds places, the thousands place and so on. If you have trouble lining up the numbers, use grid paper when adding whole numbers as shown in example 3. Also in the problems below the name of the place values are shown but students are not expected to actually write in the place value names when adding whole number.

Example 3 Evaluate 3241 + 738

	thousands	hundreds	tens	ones
	3	2	4	1
+		7	3	8
	3	9	7	9

Example 4 Evaluate 2053 + 43,741

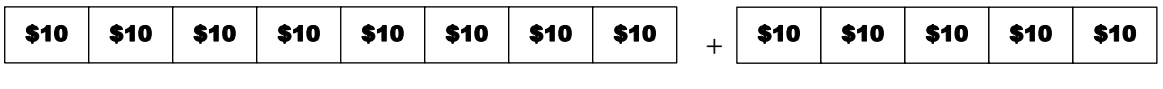
	ten thousands	thousands	hundreds	tens	ones
		2,	0	5	3
+	4	3,	7	4	1
	4	5,	7	9	4

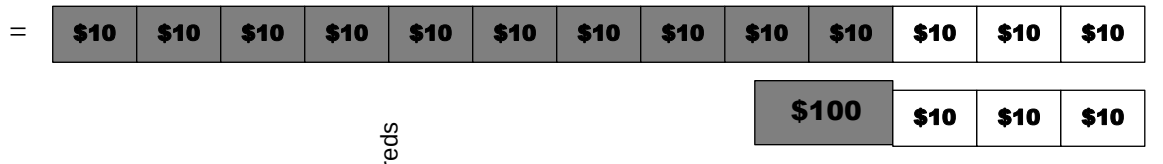
When adding whole numbers the digits in same place values are added. What happens when addition of digits in same place value result in a number larger than 9? To visualize this consider the base ten place value system in term of money. Suppose that the currency bills are limited to place value numbers.

$1 $10 $100 $1000 …

To add $80 and $50, combine the 8 ten dollar bills with 5 ten dollar bills to obtain a total of 13 ten dollar bills. Since the 13 is too large to be a digit exchange (**carry**) 10 of the ten dollar bills for 1 hundred dollar bill. So the 13 ten dollar bills are exchanged for 1 hundred dollar bill and 3 ten dollar bills.

8 tens + 5 tens = 13 tens = 1 hundreds + 3 tens





	hundreds	tens	ones
	1		
		8	0
+		5	0
	1	3	0

To add whole numbers

To add whole numbers line up their place values and add the digits of the same place value numbers. Start at the right by adding the digits in the ones place, then the tens place, the hundreds places, and so on. If at any place value the addition of the digits is larger than 9 then exchange (**carry**) 10 for 1 digit of the next successive larger place value number.

Example 5 Evaluate 473 + 294

$$\begin{array}{rccc} & \mathbf{1} & & \\ & 4 & 7 & 3 \\ + & 2 & 9 & 4 \\ \hline & 7 & 6 & 7 \end{array}$$

Example 6 Evaluate 2345 + 938

$$\begin{array}{rcccc} & \mathbf{1} & & \mathbf{1} & \\ & 2 & 3 & 4 & 5 \\ + & & 9 & 3 & 8 \\ \hline & 3 & 2 & 8 & 3 \end{array}$$

This process of adding two whole numbers can be extended to three or more whole numbers. Start at the right by adding the digits in the ones place, then the tens place, the hundreds places, the thousands place and so on and carry as necessary.

Example 7 Find the sum of 826, 543, 29, and 73

$$\begin{array}{rcccc} & & \mathbf{1} & \mathbf{2} & \\ & & 8 & 2 & 6 \\ & & 5 & 4 & 3 \\ & & & 2 & 9 \\ + & & & 7 & 3 \\ \hline & 1 & 4 & 7 & 1 \end{array}$$

An operation is **commutative** if changing the **order** in which the numbers appear does not alter the outcome of the operation. Is the operation of addition commutative? For example, if you purchase two items, detergent and orange juice, at a grocery store the total cost is the same if the price of the detergent is scanned first then the orange juice or if the price of the orange juice is scanned first and then the detergent. Consider the sum of 5 and 3 and notice that the sum is still 8 when the order in which the addends 5 and 3 are written is changed.

$$5 + 3 = 3 + 5$$

The **commutative property of addition** allows the **order** of the addends in a sum to change without altering the outcome of the sum. For any two whole number a and b, $a + b = b + a$

An operation is **associative** if changing the **grouping** of the numbers does not alter the outcome of the operation. Is the operation of addition associative? Find the sum 1, 3, and 5 and see what happens when the addends are grouped from right to left instead of from left to right as shown below? Thus, the way in which addends are grouped does not alter the outcome of a sum.

$$(1 + 3) + 5 = 1 + (3 + 5)$$
$$4 + 5 = 1 + 8$$

The **associative property of addition** allows the **grouping** of the addends in a sum to change without altering the outcome of the sum. For any whole numbers a, b and c, $(a + b) + c = a + (b + c)$

The operation of addition is both commutative and associative which means that in a sum the order and grouping of the addends can be changed. When adding a list of numbers the commutative property allows the addends to be written in any order while the associative property allows the numbers when listed vertically to be added either from the bottom to the top number or from the top to the bottom number. This allows for flexibility when adding a list of numbers as shown below.

Example 8 Find the sum of 18, 23, 45 and 12

$18 + 23 + 45 + 12 = (18 + 12) + (23 + 45) = 30 + 68 = 98$

To find the total, it is easier to reorder and regroup the numbers so that 18 and 12 are next to each other which can be added mentally.

To solve application problems requires careful reading to determine which operations are needed to solve the problem. Since addition combines addends to form a total, applications problems involving creating a total often involve sums. In the following **total problems** the addends are given and the operation of addition is used to create a sum (total) which models the problem.

Example 9 Al and Mel share the driving on a one day car trip. Al drives 134 miles and Mel drives 172 miles. How many miles were driven in the car trip?

This is a **total problem** since the unknown quantity is the total miles driven during the car trip. To find the miles driven the given addends 134 miles and 172 miles are added as shown below. The distance of the car trip is 306 miles.

$$\begin{array}{rccc} & \mathbf{1} & & \\ & 1 & 3 & 4 \\ + & 1 & 7 & 2 \\ \hline & 3 & 0 & 6 \end{array}$$

Example 10 Sharon is remodeling her kitchen which needs a new dishwasher, stove, and refrigerator. At the Appliance Warehouse she purchases a dishwasher for \$514, a stove for \$725, and a refrigerator for \$978. How much did Sharon spend on the three kitchen appliances?

This is a **total problem** since the unknown quantity is the cost of the three appliances. To find how much Sharon spends the given addends \$514, \$725, and \$978 are added as shown below. The new appliances cost a total of \$2,217.

$$\begin{array}{rcccc} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \\ & & 5 & 1 & 4 \\ & & 7 & 2 & 5 \\ + & & 9 & 7 & 8 \\ \hline & 2 & 2 & 1 & 7 \end{array}$$

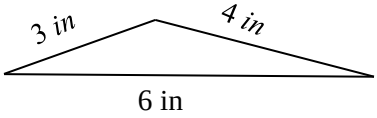
Example 11 At a local retailer, sales clerks earn \$25,000 per year and managers earn \$14,000 more than sales clerks. How much does a manager earn?

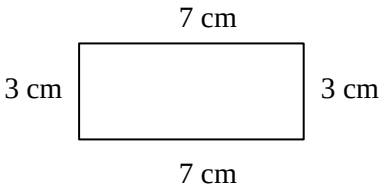
The unknown quantity is the amount a manager earns which is \$14,000 **more than** the \$25,000 earned by a sales clerk. To find a manager's annual salary the given addends \$25,000 and \$14,000 are added as shown below. A manager at this retailer earns \$39,000.

$$\begin{array}{rcccccc} & 2 & 5, & 0 & 0 & 0 \\ + & 1 & 4, & 0 & 0 & 0 \\ \hline & 3 & 9, & 0 & 0 & 0 \end{array}$$

The **perimeter** of a triangle or rectangle is calculated by adding the lengths of each of the line segments which are connected to form the shape.

Example 12 Find the perimeter of the following triangle and rectangle.



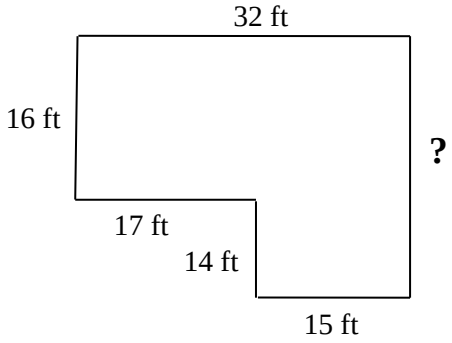


To find the perimeter of this triangle 13 inches, add the length of the sides 3, 4 and 6.

To find the perimeter of this rectangle 20 centimeters, add the length of the sides 3, 7, 3 and 7 as shown below using the associative properties to group the numbers. In a later section, a formula for the perimeter of a rectangle is given.

$$3 + 7 + 3 + 7 = (3 + 7) + (3 + 7) = 10 + 10 = 20$$

Example 13 Find the length of the missing side and then find the perimeter of the following figure.



To find the length of this right hand side which is a vertical side, the two smaller vertical sides of length 16 and 14 feet are added. The length of the missing right hand side is 30 feet

To find the perimeter of this figure sum the lengths of the six sides by adding 32, 16, 17, 14, 15 and 30. Using the commutative and associative properties these addends can be ordered and grouped in whichever way make the calculation easier to do as shown below. The perimeter of this figure is 124 feet.

$$\begin{aligned} 32 + 16 + 17 + 14 + 15 + 30 &= (16 + 14) + 30 + 32 + (15 + 17) \\ &= 30 + 30 + 32 + 32 \\ &= 124 \end{aligned}$$

Exercises 1.3

1-2 Identify the addends and the sum in the following problems.

1. $5 + 3 = 8$

2. $9 + 3 = 12$

3-8 Translate the following into numerical expressions.

3. The sum of 4 and 9
4. 8 more than 3
5. 5 increased by 2
6. Total of 7 and 4
7. 6 greater than 3
8. 9 plus 5

9-22 Evaluate the following sums.

9. 217 + 781
10. 573 + 325
11. 125 + 63
12. 7213 + 674
13. 795 + 676
14. 983 + 758
15. 76 + 248
16. 925 + 1247
17. 12,395 + 8748
18. 14,981 + 23,617
19. 85 + 123 + 74
20. 175 + 234 + 918
21. 76 + 18 + 49 + 104 + 23
22. 12,430 + 7290 + 981

23-26 Are the following true because of the commutative or the associative property? Explain in complete sentence form.

23. $5 + 6 + 7 = 5 + 7 + 6$
24. $(2 + 4) + 6 = 2 + (4 + 6)$
25. $\square + (\Delta + \¤) = (\square + \Delta) + \¤$
26. $\square + \Delta + \¤ = \square + \¤ + \Delta$

27-30 Use the commutative and associative properties to order and group the addends in the following sums to make the calculation easier.

27. 44 + 11 + 16
28. 18 + 23 + 7
29. 12 + 7 + 13 + 8
30. 11 + 6 + 13 + 19

31-36 Solve the following application problems. Show the calculations.

31. Naomi's checking account balance was $900 on Monday. During the week, she made two deposits one for $423 and another for $87 and no withdrawals. Find Naomi's balance at the end of the week.

32. Sky's Bakery earned $35,000 in profits in last year and $28,000 in profits this year. Find Sky's Bakery profits over the last two year.

33. In 2012, the three largest cities in Solano County are Vallejo with 115,928 residents, Fairfield with 106,379 residents and Vacaville with 92,092 residents. Find the population of these three cities combined.

34. Tia purchases a biology textbook for $123, an English textbook for $79, and an algebra textbook for $98. Find the total that Tia spends on textbooks.

35. The annual amount of renewable energy in gigawatts produced in California in 2012 by type is listed below. Find the total amount of renewal energy produced in California in 2012.

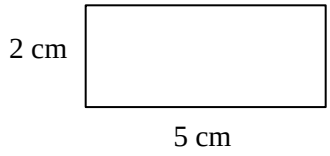
Biomass	Geothermal	Solar	Wind	Small Hydro
6031	12733	1834	9152	4257

36. The population of the three largest countries in North America in 2013 is listed below. Find the total population in the three countries combined.

Canada	Mexico	U.S.
35,236,000	118,419,000	316,102,000

37-40 Find the perimeters of the following rectangles and triangles.

37.



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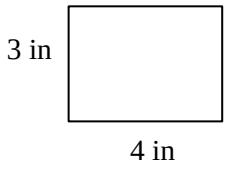

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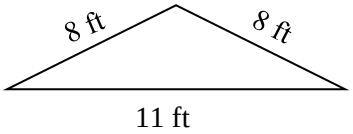

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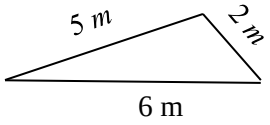
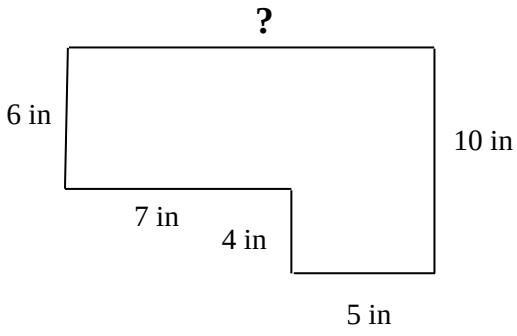


41-42 Find the length of the missing sides and perimeters of the following shapes.

41.



42.