Section 1.4 Subtracting Whole Numbers

An arithmetic operation is a process that combines quantities to create a new quantity. Subtraction is the arithmetic operation denoted by the symbol – that acts as the inverse operation of addition. The process of addition combines two given amounts to form a total, while the process of subtraction determines the amount remaining after a given amount is removed from a given total.

$$2 + 5 = 7 \quad \text{while} \quad 7 - 2 = 5$$

> The **difference** of the numbers a and b written $a - b$ is formed by subtracting the second number b, the subtrahend, from the first number a, the minuend.

There are various key words that can be used to indicate subtractions. Below 7 – 5 is written using some of the key words.

The **difference** of 7 and 5

5 **subtracted from** 7

7 **decreased** by 5

5 **less than** 7

7 **take away** 5

5 **taken away** from 7

\$5 **withdrawn** from an account with a balance of \$7

Example 1 Identify the minuend, subtrahend, and difference in 9 – 3 = 6

9	–	**3**	=	**6**
minuend		subtrahend		difference

9 is the minuend, 3 is the subtrahend, and 6 is the difference.

Example 2 Convert the following into numerical expressions.

The difference of 9 and 7	9 – 7	3 taken away from 8	8 – 3
18 decreased by 14	18 – 14	12 less than 17	17 – 12
22 take away 9	22 – 9	5 subtracted from 19	19 – 5
\$50 withdrawn from an account with a balance of \$836			836 – 50

To check the solution to a subtraction problem, a **related addition problem** is written and evaluated to make sure that the subtraction works. To check the subtraction problem $9 - 3 = 6$ the related addition problem $6 + 3 = 9$ is formed by adding the difference 6 with the subtrahend 3 to obtain the minuend 9.

Example 3 Write a related addition problem to check that $8 - 5 = 3$

$3 + 5 = 8$ can be used to check the subtraction problem $8 - 5 = 3$

To subtract whole numbers, the two numbers are written with their place values lined up and the digits with same place values are subtracted. Start at the right by subtracting the digits in the ones place, then the tens place, the hundreds places, the thousands place and so on. If you have trouble lining up the numbers, use grid paper when subtracting whole numbers as shown in the example below. Also in the example below, the name of the place values are shown but students are not expected to actually write in the place value names when subtracting.

Example 4 Evaluate $547 - 421$ and show check

	hundreds	tens	ones
	5	4	7
–	4	2	1
	1	2	6

	hundreds	tens	ones
	1	2	6
+	4	2	1
	5	4	7

Example 5 Evaluate $5839 - 607$ and show check

	5	8	3	9
–		6	0	7
	5	2	3	2

	5	2	3	2
+		6	0	7
	5	8	3	9

When subtracting whole numbers the digits in the same place values are subtracted. What happens when subtraction of digits results in a larger digit being subtracted from a smaller digit? It might be easier to visualize this by thinking of the place value numbers in term of money. Suppose that the currency bills are limited to place value numbers as shown below.

$1 $10 $100 $1000 …

Now to calculate $22 – $4, first line up the numbers and starting at the ones place four $1 bills need to be taken away from two $1 dollar bills. To resolve this issue, one $10 bill is exchanged (**borrowed**) for ten $1 bills shown below.

	tens	ones
	1	
	~~2~~	$^{1}2$
–		4
	1	8

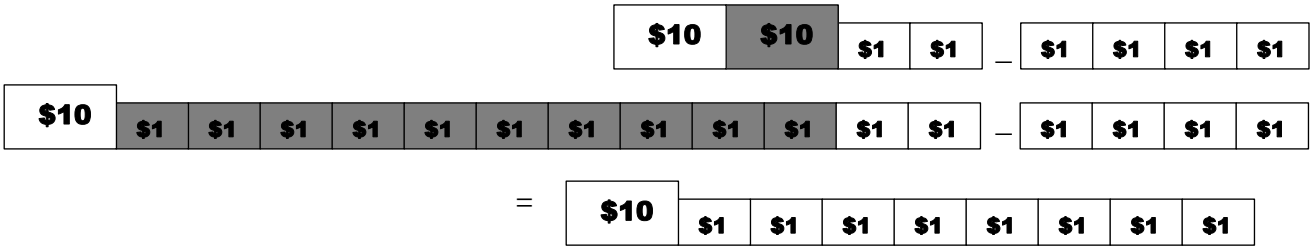
$22 – $4

= **2 tens** + 2 ones – 4 ones

= **1 tens + 10 ones** + 2 ones – 4 ones

= 1 tens + 12 ones – 4 ones

= 1 tens + 8 ones



To subtract whole numbers

To subtract whole numbers line up their place values and subtract the digits of the same place value numbers. Start at the right by subtracting the digits in the ones place, then the tens place, the hundreds places, and so on. If at any place value the subtraction of the digits results in taking away a larger digit from a smaller digit then exchange (**borrow**) 1 digit of next larger place value number for 10 units of that place value number.

Example 6 Evaluate 767 – 294 and show check

	6		
	~~7~~	$^{1}6$	7
–	2	9	4
	4	7	3

	1		
	4	7	3
+	2	9	4
	7	6	7

When subtracting above borrow one $100 bill and exchange for ten $10 bills

Example 7 Evaluate 43,283 – 2938 and show check

$$\begin{array}{r} \overset{}{}\overset{\mathbf{2}}{\cancel{3}}\overset{}{,}\overset{\mathbf{1}}{2}\overset{\mathbf{7}}{\cancel{8}}\overset{\mathbf{1}}{3} \\ -\quad 2,938 \\ \hline 40,345 \end{array} \qquad \begin{array}{r} \overset{\mathbf{1}}{4}\overset{}{0},\overset{\mathbf{1}}{3}45 \\ +\quad 2,938 \\ \hline 43,283 \end{array}$$

When borrowing from the next larger place value, a potential problem arises when the digit in the next larger place value is a zero. Unable to borrow from a zero digit in the adjacent larger place value it is then necessary to borrow from the first larger place value with a non-zero digit as shown in the following two problems.

Example 8 Evaluate 507 – 109 and show check

$$\begin{array}{r} \overset{\mathbf{4}}{\cancel{5}}\overset{\mathbf{9}}{\cancel{0}}\overset{\mathbf{1}}{7} \\ -\ 109 \\ \hline 398 \end{array} \qquad \begin{array}{r} \overset{\mathbf{1}}{3}\overset{\mathbf{1}}{9}8 \\ +\ 109 \\ \hline 507 \end{array}$$

In this problem, borrowing was needed at the ones place but since the digit in the tens place is a zero a hundred was borrowed. This borrowing from the hundreds place to the ones place is illustrated below.

5 **hundreds** = 4 **hundreds** + 10 **tens** = 4 hundreds + 9 **tens** + 10 **ones**

Example 9 Evaluate 4000 – 473 and show check

$$\begin{array}{r} \overset{\mathbf{3}}{\cancel{4}}\overset{\mathbf{9}}{\cancel{0}}\overset{\mathbf{9}}{\cancel{0}}\overset{\mathbf{1}}{0} \\ -\quad 473 \\ \hline 3527 \end{array} \qquad \begin{array}{r} \overset{\mathbf{1}}{3}\overset{\mathbf{1}}{5}\overset{\mathbf{1}}{2}7 \\ +\quad 473 \\ \hline 4000 \end{array}$$

In this problem, borrowing was needed at the ones place but since the digits in both the tens and hundreds place are zeros a thousand was borrowed. This borrowing from the thousands place to the ones place is illustrated below. Notice how borrowing from a 0 digit results in a 9 digit in that place.

4 **thousands** = 3 **thousands** + 10 **hundreds**

4 **thousands** = 3 thousands + 9 **hundreds** + 10 **tens**

4 **thousands** = 3 thousands + 9 hundreds + 9 **tens** + 10 **ones**

Applications involving subtraction can be thought of as take away or difference problems. In **take away** applications the total and a part are given with the missing part found by subtracting (taking away) the given part from the given total. In **difference** applications two numbers are given and the problem is solved by finding the difference (the gap in size) between the larger and smaller number. In some cases the same subtraction problem can be interpreted as both a take away and a difference problem. It is not important in which of the two ways the subtraction application problem is interpreted, the key is that that it is identified as a subtraction problem.

Example 10 Al and Mel share the driving on a one day car trip. The car trip is 306 miles and Al drives 134 miles. How many miles did Mel drive?

The unknown quantity is the miles driven by Mel, while the total length of the car trip 306 miles and the part of the trip driven by Al 134 miles are given. This problem can be categorized as a **take away** problem with 134 miles driven by Al subtracted from the total of 306 miles driven as shown below. Mel drove for 172 miles

$$\begin{array}{rrrr} & \overset{\mathbf{2}}{} & & \\ & \cancel{3}\,{}^{\mathbf{1}}0 & 6 & \\ - & 1 & 3 & 4 \\ \hline & 1 & 7 & 2 \end{array}$$

Example 11 Sam's Automotive earned \$75,000 in profits in last year and \$68,000 in profits this year. How much less did Sam's Automotive earn in profits this year?

The unknown quantity is how much less is earned in profits this year as compared to last year. This problem can be categorized a **difference problem** which is solved by finding the difference (gap) between last year's profit \$75,000 and this year's profit \$68,000. As shown below, Sam's Automotive earned \$7000 less in profits this year.

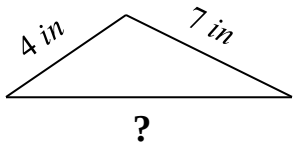
$$\begin{array}{rrrrrr} & \overset{\mathbf{2}}{} & & & & \\ & \cancel{7}\,{}^{\mathbf{1}}5, & 0 & 0 & 0 & \\ - & 6 & 8, & 0 & 0 & 0 \\ \hline & & 7, & 0 & 0 & 0 \end{array}$$

Example 12 Lola is currently 21 years old and has a younger brother Phil who is 15 years old. How many years older is Lola than her brother Phil?

The unknown quantity is how many years older Lola is than Phil. This problem can be interpreted as a **difference problem** solved by finding the difference (gap) in ages between the older Lola 21 and younger Phil 15. This problem can also be interpreted as a **take away** problem with Phil's age of 15 taken away from Lola's age of 21. Either way, Lola is currently 6 years older than Philip, since 21 – 15 equals 6.

For some application problems multiple operations are required to solve the problem. In the following applications multiple operations involving addition and subtraction are used to solve the problems.

Example 13 Find the missing side of the triangle given the perimeter is 20 inches



To find the unknown quantity, the length of the bottom side of the triangle, first add the lengths of the two given sides of the triangle 4 and 7 inches and then subtract that amount from the length of the given perimeter 20 inches as shown below. The length of the bottom side of the triangle is 9 inches.

$$4 + 7 = 11 \quad \text{then} \quad 20 - 11 = 9$$

The solution shown above involved one addition and one subtraction. This problem can also be solved by using two subtractions, first subtracting 4 inches from the 20 inches perimeter then subtracting the 7 inches from that difference.

$$20 - 4 = 16 \quad \text{then} \quad 16 - 7 = 9$$

Example 14 Jill budgeted $2000 for a dishwasher, stove, and refrigerator. She purchased the dishwasher for $500 and stove for $700, how much does Jill have left in the budget for a refrigerator.

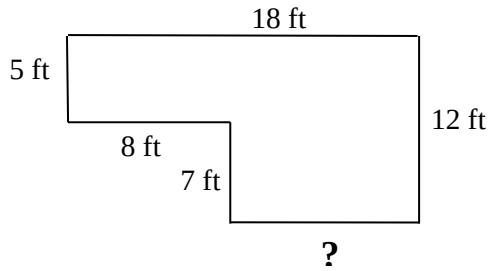
To find the unknown quantity, the funds left to buy a refrigerator, first find how much has been spend by adding the addends $500 and $700 to obtain $1200 and then **take away** the $1200 spend from the budget of $2000. Jill has $800 left.

$$500 + 700 = 1200 \quad \text{then} \quad 2000 - 1200 = 800$$

The solution shown above involved one addition and one subtraction. This problem can also be solved by using two subtractions, first subtracting $500 from the $2000 budget, then subtracting the $700 from that difference.

$$2000 - 500 = 1500 \quad \text{then} \quad 1500 - 700 = 800$$

Example 15 Find the length of the missing side and then find the perimeter of the following figure.



To determine the length of the bottom side which is a horizontal side, look at the two other horizontal sides of length 18 and 8 feet. The missing bottom side is found by taking the difference of 18 feet the larger horizontal side and 8 feet the smaller horizontal side. The length of the missing bottom side is 10 feet.

$$18 - 8 = 10$$

To find the perimeter of this figure sum the lengths of the six sides by adding 18, 5, 8, 7, 10 and 12. Using the commutative and associative properties these addends can be ordered and grouped in whichever way make the calculation easier to do as shown below. The perimeter of this figure is 60 feet.

$$18 + 5 + 8 + 7 + 10 + 12 = (18 + 12) + 10 + (5 + 8 + 7) = 30 + 10 + 20 = 60$$

An operation is **commutative** if changing the **order** in which the numbers appear does not alter the outcome of the operation. Is the operation of subtraction commutative? No, consider the sum 5 and 3 and notice that the sum is different when the order in which the values 5 and 3 appear are written is changed.

$$5 - 3 \neq 3 - 5$$

An operation is **associative** if changing the **grouping** of the numbers does not change the outcome of the operation. Is the operation of subtraction associative? No, consider the difference involving the numbers 7, 5, and 1 and see what happened when the numbers are grouped from right to left instead of from left to right?

$$(7 - 5) - 1 \neq 7 - (5 - 1)$$

$$2 - 1 \neq 7 - 4$$

The operation of addition is both commutative and associative which means that in a sum the order and grouping of the addends can be changed. This allows for flexibility when adding a list of numbers. But the operation of **subtraction** is **neither commutative nor associative** which means that in subtraction calculations the order and grouping cannot be changed.

Exercises 1.4

1-2 Identify the minuend, subtrahend, and difference in the following problems.

1. $5 - 3 = 2$

2. $9 - 3 = 6$

3-14 Translate the following into numerical expressions.

3. Difference of 8 and 3
4. 6 more than 4
5. 3 increased by 5
6. 9 decreased by 7
7. 8 subtracted from 12
8. 9 plus 3
9. 4 greater than 8
10. 7 taken away from 11
11. 4 less than 9
12. The sum of 5 and 2
13. Total of 3 and 6
14. 15 take away 4

15-18 Write a related addition problem to check that the following subtractions.

15. $9 - 2 = 7$
16. $6 - 1 = 5$
17. $30 - 12 = 18$
18. $24 - 18 = 6$

19-32 Evaluate the following subtractions. **Show check for each problem**.

19. $439 - 207$
20. $895 - 743$
21. $125 - 63$
22. $213 - 176$
23. $1245 - 893$
24. $2053 - 459$
25. $12{,}317 - 2481$
26. $34{,}162 - 25{,}075$
27. $400 - 196$
28. $503 - 379$
29. $2089 - 1983$
30. $2007 - 1806$
31. $5000 - 3791$
32. $10{,}000 - 4573$

33-40 Solve the following application problems. Show the calculations.

33. Solano College had 93 math courses in fall 2011 and 85 math courses in spring 2012. How many more math courses were offered in fall 2011 then spring 2012 at Solano College?

34. The 2014 payroll for the Oakland A's is \$74,765,900 and for the San Francisco Giants is \$147,738,612. How much larger is the Giants' payroll?

35. In a math class that starts with 38 students, 26 students finished the course with a passing grade. How many students did not pass the course?

36. A regularly priced $237 smartphone is on sale for $189. How much is the smartphone discounted during the sale?

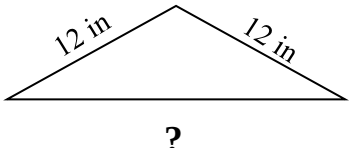
37. In 2012, the three largest cities in Solano County are Vallejo with 115,928 residents, Fairfield with 106,379 residents and Vacaville with 92,092 residents. The entire population of Solano County is 413,786 residents, how many Solano County residents do not live in Vallejo, Fairfield or Vacaville?

38. Jill's checking account balance was $1428 on Friday. During the weekend, she made two withdrawals one for $80 and another for $715 and a deposit of $372. What is Jill's checking account balance on Monday morning?

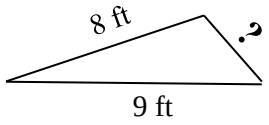
39. The Washington family has a monthly budget of $3100. If they spend $1200 on rent, $520 on food, $340 on utilities, $410 on transportation how much is left over for other expenses?

40. For new furniture, Genele has budgeted $2100. If she spends $725 on a couch, $327 on a coffee table and $436 on a love seat, how much does she still have left to purchase other items?

41. Find the missing side of the following triangle whose perimeter is 40 inches.

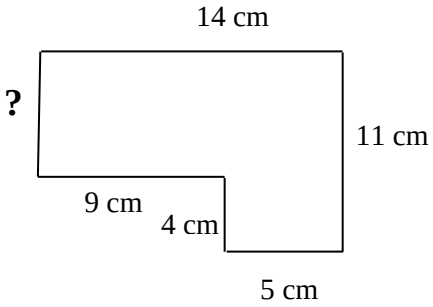


42. Find the missing side of the following triangle whose perimeter is 20 feet.



43-44 Find the length of the missing sides and perimeters of the following shapes.

43.



44.