Section 1.5 Multiplying Whole Numbers

Multiplication is the arithmetic operation denoted by the symbol $\times$ that determines the total resulting from repeatedly adding the same number. If you purchase 12 cans of same size and brand of cat food at a grocery store instead of scanning all the cans, the cashier can shortcut the process by scanning the price of one can of the cat food and multiplying that amount by 12. The multiplication operation determines the total formed by a given number of equal sized groups. Thus, the multiplication operation serves as shortcut for repeated addition of the same number. As illustrated below the product 4×2 represents 4 groups of 2 smiley faces which when written in **expanded addition form** equals $2 + 2 + 2 + 2$.

Example 1 Write the expanded addition form of the products 3×7 and 5×6

3×7 represents three groups of seven $\quad 3 \times 7 = 7 + 7 + 7$

5×6 represents five groups of six $\quad 5 \times 6 = 6 + 6 + 6 + 6 + 6$

The word **product** denotes multiplication with **factors** being the numbers multiplied. For instance, $4 \times 5 = 20$ has factors 4 and 5 and the product is 20. Multiplication can also be denoted using a vertically centered dot between the factors or by listing one or both of the factors in parenthesis as shown below.

$$4 \times 5 = 4 \cdot 5 = (4)(5) = 4(5) = (4)(5)$$

Example 2 Identify the factors and products in the following multiplications.

$6 \times 4 = 24$ Factors are 6 and 4 and the product is 24

$(5)(7) = 35$ Factors are 5 and 7 and the product is 35

To list the multiples of a number start with the given number and then multiply the given number by 1, 2, 3, … Multiples of a given number are sums with addends consisting solely of the given number. Below, some multiples of 4 are generated.

$$1 \times \mathbf{4} = 1 \text{ group of } \mathbf{4} = \mathbf{4} = 4$$
$$2 \times \mathbf{4} = 2 \text{ groups of } \mathbf{4} = \mathbf{4} + \mathbf{4} = 8$$
$$3 \times \mathbf{4} = 3 \text{ groups of } \mathbf{4} = \mathbf{4} + \mathbf{4} + \mathbf{4} = 12$$
$$4 \times \mathbf{4} = 4 \text{ groups of } \mathbf{4} = \mathbf{4} + \mathbf{4} + \mathbf{4} + \mathbf{4} = 16$$
$$5 \times \mathbf{4} = 5 \text{ groups of } \mathbf{4} = \mathbf{4} + \mathbf{4} + \mathbf{4} + \mathbf{4} + \mathbf{4} = 20$$

To create a multiplication table the first nine multiples of the digits 1 through 9 are listed both horizontally and vertically as shown below. The digit 0 is not listed since multiplication by digit 0 always results in 0.

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Example 3 Evaluate the following products: $8 \cdot 7$ $(9)(8)$ 0×5 $(1)(6)$

$8 \cdot 7 = 56$ $(9)(8) = 72$

$0 \times 5 = 0$ $(1)(6) = 6$

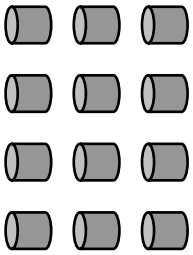
Example 4 Find the total number of seats in a restaurant that has 9 tables with 4 seats each and 5 booths with 6 seats each

The solution to this problem is the total number of seats in this restaurant. There are two separate multiplications involved, the product 9×4 that determines the number of seats in 9 tables with 4 seats each and 5×6 which determines the number of seats in 5 booths with 6 seats per booth. Then the number of seats in the 9 tables and the 5 booths are added as shown below. This restaurant has 66 seats.

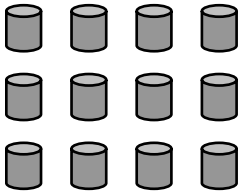
9 tables with 4 chairs per table	$9(4) = 36$
5 booths with 6 chairs per table	$5(6) = 30$
Total chairs in restaurant	$36 + 30 = 66$

An operation is **commutative** if changing the **order** in which the numbers appear does not alter the outcome of the operation. Is the operation of multiplication commutative? Consider the product of 4 and 3 and notice that the product remains the same when the order in which the factors 4 and 3 are written is changed with 4(3) equal to 3(4). This means that 4 groups of 3 are equal to 3 groups of 4. This is illustrated below by drawing 4 groups of 3 cans, then rotating the drawing so that the figure represents 3 groups of 4 cans with both figures containing a total of 12 cans.

The product (4)(3) is the total formed by 4 (rows) groups of 3 cans which equals 12 cans.



The product (3)(4) is the total formed by 3 (rows) groups of 4 cans which equals 12 cans.



The **commutative property of multiplication** allows the **order** of the factors in a product to change without altering the outcome of the product. For any two numbers a and b, $a \times b = b \times a$

An operation is **associative** if changing the **grouping** of the numbers does not alter the outcome of the operation. Is the operation of multiplication associative? Find the product with the factors 2, 3, and 4 and see what happens when the factors are grouped from right to left instead of from left to right as shown below? In either case the product is 24. Thus, the way in which factors are grouped does not alter the outcome of a product.

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$
$$6 \times 4 = 2 \times 12$$

The **associative property of multiplication** allows the **grouping** of the factors in a product to change without altering the outcome of the product. For any whole numbers a, b and c, $(a \times b) \times c = a \times (b \times c)$

The operation of multiplication is both commutative and associative which means that in a product the order and grouping of the factors can be changed. This allows for flexibility when multiplying a list of numbers.

To develop a technique to multiply a one digit number times a multi digit number think of multiplication as repeated addition of the same number. Below, the product 4×873 in is written in the expanded addition form as 4 groups of 873.

$$\begin{array}{rcccc} & & \mathbf{2} & \mathbf{1} & \\ & & 8 & 7 & 3 \\ & & 8 & 7 & 3 \\ & & 8 & 7 & 3 \\ + & & 8 & 7 & 3 \\ \hline & 3 & 4 & 9 & 2 \end{array}$$

Now the product 4×873 is calculated in the traditional multiplication method by starting at the right and multiplying the 4 times the digit 3 in the ones place, then 4 times the 7 in the tens place, and finally 4 times the 8 in the hundreds place and carrying as necessary. Notice how the **1** carried to the tens place and the **2** carried to the hundreds place are the same when the calculation is done either by repeated addition of the same number or by multiplying.

$$\begin{array}{rcccc} & & \mathbf{2} & \mathbf{1} & \\ & & 8 & 7 & 3 \\ \times & & & & 4 \\ \hline 3 & 4 & 9 & 2 & \end{array}$$

4×3 ones = 12 ones = 1 tens + 2 ones
Carry 1 ten and put 2 in the ones place.

4×7 tens + **1** tens = 28 tens + **1** tens = 29 tens = 2 hundreds + 9 tens
Carry 2 hundreds and put 9 in the tens place

4×8 hundreds + **2** hundreds = 32 hundreds + **2** hundreds = 34 hundreds = 3 thousands + 4 hundreds
Put 3 in the thousands and 4 in the hundreds place

To multiply a one digit number times a multi digit number

Start at right and multiply the one digit number times the digit in the ones places of the larger number and carry if necessary, then multiply the one digit number times the digit in the tens place of the larger number and carry as necessary. Repeat this process until the one digit number is multiplied times each digit of the larger number.

Example 5 Evaluate 9×48

$$\begin{array}{rcc} & \mathbf{7} & \\ & 4 & 8 \\ \times & & 9 \\ \hline 4 & 3 & 2 \end{array}$$

9×8 ones = 72 ones = 7 tens + 2 ones
Carry 7 tens and put 2 in the ones place.

9×4 tens + **7** tens = 36 tens + **7** tens = 43 tens = 4 hundreds + 3 tens
Put the 4 in the hundreds place and 3 in the tens place

Before multiplying multi digit numbers, consider products with contain a factor that is a multiple of 10. For instance 8×400 has a factor 400 which is a multiple of 10. For multiplications involving multiples of 10 a shortcut technique is available which serves as an alternative to standard multiplication. To evaluate 8×400 simply multiply the non-zero digits 8 and 4 and then insert the two zero digits at the end with the product 8×4**00** equaling 32**00.**

For multiplications involving factors which end in a zero digit (multiples of 10) simply multiply the leading non-zero digits and insert at the right the total number of zeroes at the end of both factors.

Example 6 Evaluate the following products: $15 \times 10{,}000$ (7)(900) (60)(300)

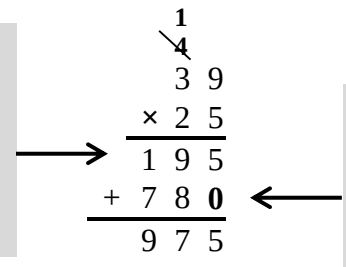
15×1**0,000** = 15**0,000**
Multiply the nonzero digits 1 and 15 and insert four zeroes at the right

(7)(9**00**) = 6,3**00**
Multiply the nonzero digits 7 and 9 and insert two zeroes at the right

(6**0**)(3**00**) = 18,**000** Note both of these numbers are multiples of 10
Multiply the nonzero digits 6 and 3 and insert three zeroes at the right

To multiply multi digit numbers write one of the factors in expanded form. For instance to evaluate 39×25 first write 25 in expanded addition form as 20 + 5 and then break the product 39×25 into two products 39×20 and 39×5 as shown below.

First multiply 5 times 39
$5 \times 9 = 45$
Carry 4 and put 5 in the ones place
$5 \times 3 + \mathbf{4} = 15 + \mathbf{4} = 19$
Put 1 and 9 in the next two places
Cross out the carried amounts



Then multiply 2**0** times 39
Put **0** in the ones place on 2nd line
Now multiply 2 times 39
$2 \times 9 = 18$
Carry 1 and put 8 in the next place
$2 \times 3 + \mathbf{1} = 6 + \mathbf{1} = 7$
Put the 7 in the next place
Add the two rows of numbers

To multiply multi digit numbers

Start at right and multiply the digit in the ones place of the smaller number times the larger number. Then start a new row with a 0 digit and multiply the digit in the tens place of the smaller number times the larger number. Repeat this process until all the digits in the smaller number are multiplied times the larger number. Then add up the rows.

Example 7 Evaluate the following products: 76×38 756×823 402×529

First multiply 8 times 76 and enter the product on the first line. Then multiply 30 times 76 and enter the product on the second line. Add the two lines to obtain the final answer 2888

$$\begin{array}{rl} {\scriptstyle 1\ \ 4} & \\ 7\ 6 & \\ \times\ 3\ 8 & \\ \hline 6\ 0\ 8 & (8 \times 76) \\ +\ 2\ 2\ 8\ \mathbf{0} & (\mathbf{3}0 \times 76) \\ \hline 2\ 8\ 8\ 8 & \end{array}$$

First multiply 6 times 823 and enter the product on the first line. Then multiply 50 times 823 and enter the product on the second line. Next multiply 700 times 823 and enter the product on the third line. Add the three lines to obtain the final answer 622,188

$$\begin{array}{rl} {\scriptstyle 1\ 2\ \ \ } & \\ {\scriptstyle 1\ 1\ 1} & \\ {\scriptstyle 1\ \ 1} & \\ 8\ 2\ 3 & \\ \times\ 7\ 5\ 6 & \\ \hline 4\ 9\ 3\ 8 & (6 \times 823) \\ 4\ 1\ 1\ 5\ \mathbf{0} & (\mathbf{5}0 \times 823) \\ +\ 5\ 7\ 6\ 1\ \mathbf{0}\ \mathbf{0} & (\mathbf{7}\mathbf{0}\mathbf{0} \times 823) \\ \hline 6\ 2\ 2,\ 1\ 8\ 8 & \end{array}$$

First multiply 2 times 529 and enter the product on the first line. Then multiply 400 times 529 and enter the product on the second line. Add the two lines to obtain the final answer 212,658

$$\begin{array}{rl} {\scriptstyle 1\ 3} & \\ {\scriptstyle \ \ 1} & \\ 5\ 2\ 9 & \\ \times\ 4\ 0\ 2 & \\ \hline 1\ 0\ 5\ 8 & (2 \times 529) \\ +\ 2\ 1\ 1\ 6\ \mathbf{0}\ \mathbf{0} & (\mathbf{4}\mathbf{0}\mathbf{0} \times 529) \\ \hline 2\ 1\ 2,\ 6\ 5\ 8 & \end{array}$$

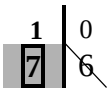
For some multiplication problems involving multi digit numbers an estimate that can be easily and quickly calculated is sufficient. One way to estimate a product quickly is to first front end round each factor then multiply those rounded factors. To **front end round** a whole number, round it to the leading digit place (the first non-zero digit on the left/front).

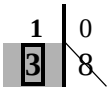
Example 8 Estimate the following products using front end rounding

$76 \times 38 \approx 8\mathbf{0} \times 4\mathbf{0} = 32\mathbf{00}$

Front end round **7**6. The leading digit is the 7 in the **tens** place 80

Front end round **3**8. The leading digit is the 3 in the **tens** place 40

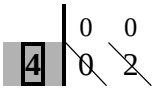


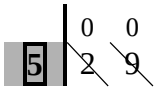


$402 \times 529 \approx 4\mathbf{00} \times 5\mathbf{00} = 20\mathbf{0{,}000}$

Front end round 402. The leading digit is the 4 in the **hundreds** place. 400

Front end round 529. The leading digit is the 5 in the **hundreds** place. 500





Multiplication is the arithmetic operation that determines the total resulting from repeatedly adding the same number. Applications that involve multiple same sized groups are modeled using multiplication. To solve application problems requires careful reading to determine which operations are needed to solve the problem. In solving the following multiplication application problems first identify the common group size and the number of groups which become the factors in the product.

Example 9 An online bookstore sells books for $20. If during the last year Tia purchased 18 books from this site, how much did she spend on books at this site last year?

The solution to this problem is the amount Tia spend on books at this site last year. The common group size is $20 per book the number of groups is 18 books. The amount spend on books by Tia is 18 groups of $20 which is modeled by (18)(20). The product (18)(2**0**) equals 360 by multiplying 18 times 2 and inserting a zero digit at the end. Tia spent $360 on bestselling books at the site last year.

Example 10 Dwayne leases a new car for \$175 per month for three years. How much does he pay in lease payments over the three year lease period?

The solution to this problem is the total lease payments paid over a three year period. First, find the total number of monthly payments made during the three year lease which is 3 groups of 12 monthly payments with (3)(12) equaling 36. The total lease payments paid is 36 monthly payments with a common group size of \$175 which is modeled by the product (36)(75). As shown below, Dwayne paid \$6300 in lease payments over the three years.

$$\begin{array}{r} 12 \\ \times\ \ 3 \\ \hline 36 \end{array}$$

$$\begin{array}{rl} & \begin{array}{r} \overset{2}{\cancel{4}}\,\overset{1}{\cancel{3}} \\ 175 \end{array} \\ \times & 36 \\ \hline & 1050 \quad (6 \times 175) \\ + & 5250 \quad (30 \times 175) \\ \hline & 6300 \end{array}$$

This problem can also be modeled by the product 3×12×175. Since multiplication is both commutative and associative, the order and grouping of the factors can be changed before evaluating this product.

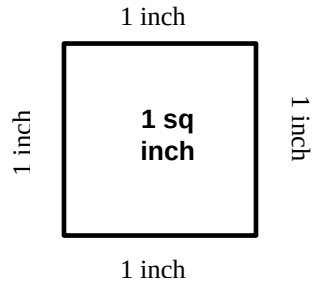
In recent years to encourage conservation of water and energy, utility companies have established tiered rates which charge more per unit for those households that consume large amounts of resources. Tiered rates are also used in federal and state income taxes rates in which higher incomes are taxed at a higher rate.

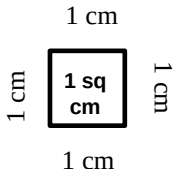
Example 11 The city water department has a tiered water rate with a cost of \$4 per thousand gallons of water for the first 10 thousand gallons and \$6 per thousand gallons of water for any thousands of gallons of water above 10 thousand gallons. How much is the monthly water bill for a household that consumes 18 thousand gallons in a given month?

The 18 thousand gallons of water consumed is broken into two pricing tiers, the first 10 thousand gallons are billed at \$4 per thousand gallons and the next 8 thousand gallons are billed at \$6 per thousand gallons which results in two products. The first product (10)(4) represents 10 groups with a common group size of \$4 per thousand gallon. While the second product (8)(6) represents 8 groups with a common group size of \$6 per thousand gallons. To find the monthly water bill, the two resulting products are evaluated separately then added as shown below. The monthly water bill for a household that consumes 18 thousand gallons of water is \$88.

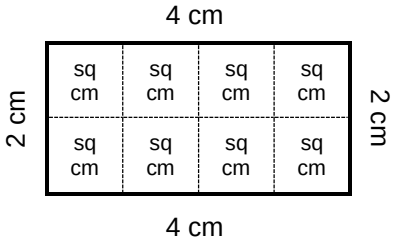
Cost of first 10 thousand gallons of water	(1**0**)(4) = 4**0**
Cost of the next 8 thousand gallons of water	(8)(6) = 48
Total cost of 18 thousand gallons of water	40 + 48 = 88

The area of a two dimensional shape is the amount of space measured in square units that the shape covers. Below is drawn, 1 square inch as a square with one inch sides and 1 square centimeter as a square with one centimeter sides.





The area of a rectangle is determined by how many square units are required to cover the entire rectangle. Below is drawn a rectangle with a length of 4 centimeters and a width of 2 centimeters. This rectangle is covered by eight of the 1 square centimeter squares, thus the area of this rectangle is 8 square centimeters. The area of this rectangle with a length of 4 and a width of 2 centimeters is 8 square centimeters which is the product of the length and the width, since 4 times 2 equals 8.



The **area of a rectangle** is the product of the length times the width of the rectangle.

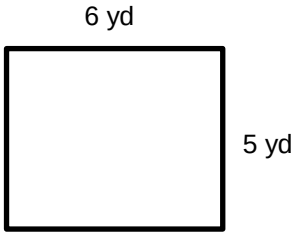
(length)(width)

Example 12 Find the area of the rectangle with length 8 feet and width 3 feet.

(8 ft)(3 ft) = 24 sq ft or 24 ft^2

This rectangle has an area of 24 square feet

Example 13 Find the area of the rectangle drawn below.



(6 yd)(5 yd) = 30 sq yd or 30 yd^2

This rectangle has an area of 30 square yards

Exercises 1.5

1-2 Identify the factors and products in the following problems.

1. $(4)(6) = 24$
2. $8 \times 5 = 40$

3-10 Translate the following into numerical expressions.

3. The product of 6 and 4
4. The sum of 8 and 3
5. The difference of 11 and 5
6. 8 groups of 3
7. 9 decreased by 1
8. 4 increased by 6
9. 3 times 7
10. 7 taken away from 11

11-12 Are the following true because of the commutative or the associative property? Explain in complete sentence form.

11. $4 \times 5 \times 3 = 5 \times 4 \times 3$
12. $(2 \times 5) \times 8 = 2 \times (5 \times 8)$

13-18 Evaluate mentally (*no steps needed*) the following products.

13. $(8)(1)$
14. 0×9
15. $(45)(0)$
16. 1×53
17. $(4)(8)$
18. 9×7

19-30 Evaluate (*show steps*) the following products.

19. $(27)(48)$
20. 56×19
21. $(71)(32)$
22. 45×93
23. $(217)(23)$
24. 805×45
25. $(37)(109)$
26. 323×28
27. $(453)(72)$
28. 521×218
29. $(231)(405)$
30. 504×321

31-36 Evaluate the following products using the shortcut for multiplying with factors that are multiples of ten.

31. $(12)(300)$
32. 200×17
33. $(400)(60)$
34. 500×700
35. $(500)(31)$
36. 210×30

37-42 **Estimate** the following products by first front end rounding each factor and then using the shortcut for multiplying with factors that are multiples of ten.

37. $(37)(52)$
38. 64×77
39. $(48)(213)$
40. 88×217
41. $(231)(415)$
42. 524×309

43-49 Solve the following application problems. Show the calculations.

43. Jerome purchases a $25 gift card for each of his three daughters. How much does he spend on gift cards?

44. Sheila purchase a dining table for $210 and four chairs for $54 per chair. How much did she spend for the dining room table and chairs?

45. Sean estimates that he uses three gallons of gas to drive to and from work. How many gallons of gas does he consume in a five day workweek driving to and from work? If the price of gas is four dollars per gallon, estimate his weekly cost of gas to drive to and from work?

46. Makayla recently purchased a car and paid a down payment of $2500 and monthly payments of $240 per month for five years. How much will she pay in payments including the down payment during the five year period?

47. A company is expanding and needs to purchase four computers and two printers. If the computers cost $640 and the printers cost $185, how much does the company need to spend on computers and printers?

48. On a vacation car trip, Juwan drives 3 hours at an average of 65 miles per hour and then has lunch. After lunch he drives 4 hours at an average of 57 miles per hours. How far did Juwan drive during the 7 hour trip?

49. A 150 pound person that walks one mile in 20 minutes burns about 80 calories. Chandra weighs 150 pounds and walks three miles at a 20 minute per mile pace five times each week. Find the total calories that Chandra burns walking in a week.

For #50-55 write final answers with the appropriate units included

50. Find the area of the rectangle with length 9 feet and width 4 feet.

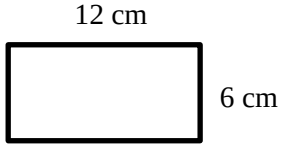
51. Find the area of the rectangle with length 7 yards and width 5 yards.

52. Find the area of a standard doubles tennis court which is a rectangle that measures 78 feet by 36 feet.

53. Find the area of a rectangular property that is 88 yards long by 55 yards wide. (*Note the area of this property is exactly one acre*)

54-55 Find the area of a rectangles drawn below.

54.



55.