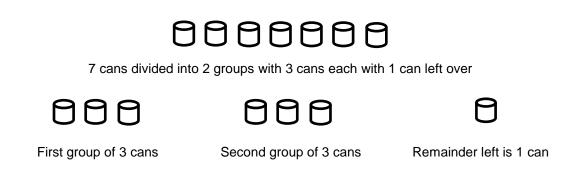**Section 1.7   Dividing Whole Numbers (Part II)**

In a division problem, if the dividend is a not a multiple of the divisor, the resulting quotient will have a remainder and the dividend is **not divisible** by the divisor. Below a total of 7 cans are placed in equal size groups with 3 cans each which results in 2 groups of 3 cans with 1 can left over as the remainder. This is modeled by the division problem $7 \div 3 = 2$ R1. To check that $7 \div 3 = 2$ R1 multiply the quotient 2 times the divisor 3 and then add the remainder 1 and verify that $2 \times 3 + 1$ equals the dividend 7.

7 cans divided into 2 groups with 3 cans each with 1 can left over

First group of 3 cans   Second group of 3 cans   Remainder left is 1 can

Multiplication creates a total formed by repeated addition of the same number (the common group size). In a similar way, division uses repeated subtraction by starting at the given total (dividend) and repeatedly subtracting the common group size until the remainder is smaller than the divisor. The division problem $7 \div 3 = 2$ R1 is evaluated on the right by starting with the total of 7 (the dividend) and subtracting by the group size (divisor) 3 repeatedly, in this case 2 times, until the remaining value 1 is smaller than the group size (divisor) 3. While the repeated subtraction technique works an efficient process is needed to evaluate quotients.

$$
\begin{array}{r}
7 \\
-\,\mathbf{3} \\
\hline
4 \\
-\,\mathbf{3} \\
\hline
1
\end{array}
$$

An efficient process to evaluate quotient is long division. Below the long division process is used to calculate $7 \div 3$.
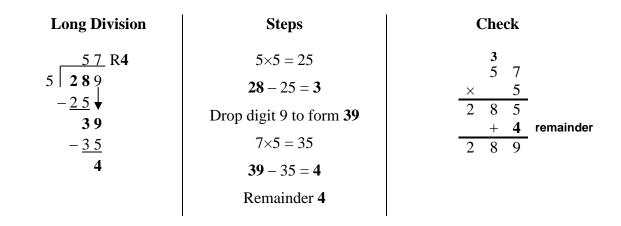
Long Division	**Steps**	**Check**
$\begin{array}{r} 2\ \mathbf{R1} \\ 3\,\overline{\big)\ 7} \\ -\,\underline{6} \\ 1 \end{array}$	$2 \times 3 = 6$ $\mathbf{7 - 6 = 1}$	$2 \times 3 + \mathbf{1} = 6 + \mathbf{1} = 7$

Long division provides a step by step process to evaluate quotients. For long division the first step is to compare the divisor with the leading digit or digits of the dividend until the divisor is smaller than or equal to the leading part of the dividend. If the divisor is smaller than or equal to the leading digit of the dividend, determine how many times the divisor goes into the leading digit of the dividend. If the divisor is larger than the leading digit of the dividend, then use two or more of the leading digits of the dividend until the divisor is smaller than or equal to the leading part of dividend and then determine who many times the divisor goes into those leading digits of the divisor. Make sure that the first digit in the quotient is correctly lined up with the last digit of leading digits of the dividend that it is divided into.   To demonstrate long division $58 \div 8$ is evaluated below by first determining how many times the divisor 8 goes into the dividend 58 such that the remainder is less than the divisor 8? It goes in seven times with $7\times 8 = 56$ which when subtracted from 58 gives a remainder of 2. To check quotients with remainders show that the product of the quotient and the divisor when added to the remainder equals the dividend.

Long Division	**Steps**	**Check**
$\begin{array}{r} 7\,\text{R}2 \\ \hline 8\,|\;5\,8 \\ -\,5\,6 \\ \hline 2 \end{array}$	$7\times 8 = 56$ $58 - 56 = \mathbf{2}$	$7\times 8 \;+\; \mathbf{2}$ $= 56 + \mathbf{2} = 58$

To evaluate $289 \div 5$ using long division first notice that the divisor 5 is greater than the leading digit 2 of the dividend 289, so as shown below start with the first two leading digits of the dividend **28**9 and determine how many times the divisor 5 goes into **28**? It goes in five times with $5\times 5 = 25$ which when subtracted from **28** leaves a remainder of **3**. Then drop down the next digit in the dividend 9 to form **39** and determine how many times the divisor 5 goes into **39**? It goes in seven times with $7\times 5 = 35$ which when subtracted from **39** the remainder is **4**.

Long Division	**Steps**	**Check**
$\begin{array}{r} 5\,7\,\text{R}\mathbf{4} \\ \hline 5\,|\;2\,8\,9 \\ -\,2\,5\;\downarrow \\ \hline 3\,9 \\ -\,3\,5 \\ \hline 4 \end{array}$	$5\times 5 = 25$ $28 - 25 = \mathbf{3}$ Drop digit 9 to form **39** $7\times 5 = 35$ $39 - 35 = \mathbf{4}$ Remainder **4**	$\begin{array}{r} \mathbf{3} \\ 5\;\;7 \\ \times\quad\;\;5 \\ \hline 2\;\;8\;\;5 \\ +\;\;\mathbf{4}\quad\text{remainder} \\ \hline 2\;\;8\;\;9 \end{array}$

Chapter One

Example 1   Use long division to evaluate $974 \div 4$ and show check

To start the long division process since the divisor 4 is smaller than the leading digit 9 of the dividend **9**74, determine how many times the divisor 4 goes into leading digit of the dividend 9? It goes in two times with $2 \times 4 = 25$, so put the 2 in the quotient directly above the 9 in the dividend. Then proceed as shown below.

Long Division	Steps	Check

Long Division

$$
\begin{array}{r}
2\,4\,3\ \text{R}2 \\
4\,\overline{)\,9\,7\,4} \\
-\,8 \\
\hline
1\,7 \\
-\,1\,6 \\
\hline
1\,4 \\
-\,1\,2 \\
\hline
2
\end{array}
$$

Steps

$2 \times 4 = 8$

$9 - 8 = 1$

Drop digit 7 to form **17**

$4 \times 4 = 16$

$17 - 16 = 1$

Drop digit 4 to form **14**

$3 \times 4 = 12$

$14 - 12 = 2$

Remainder **2**

Check

$$
\begin{array}{r}
1\ \ 1 \\
2\ 4\ 3 \\
\times \quad 4 \\
\hline
9\ 7\ 2 \\
+\ \ 2 \quad \text{remainder} \\
\hline
9\ 7\ 4
\end{array}
$$

Example 2   Use long division to evaluate $3015 \div 12$ and show check

Since the divisor 12 is larger than the leading digit 3 of the dividend 3015 use the two leading digits 30 of the dividend **30**15. Determine how many times the divisor 12 goes into the leading digits **30**. It goes in 2 times with $2 \times 12 = 24$, so put the 2 in the quotient directly above the 0 in the dividend which is the last digit in the leading digit of the dividend **30**. Then proceed as shown below.

Long Division

$$
\begin{array}{r}
2\,5\,1\ \text{R}3 \\
12\,\overline{)\,3\,0\,1\,5} \\
-\,2\,4 \\
\hline
6\,1 \\
-\,6\,0 \\
\hline
1\,5 \\
-\,1\,2 \\
\hline
3
\end{array}
$$

Steps

$2 \times 12 = 24$

$30 - 24 = 6$

Drop digit 1 to form **61**

$5 \times 12 = 60$

$61 - 60 = 1$

Drop digit 5 to form **15**

$1 \times 12 = 12$

$15 - 12 = 3$

Remainder **3**

Check

$$
\begin{array}{r}
1 \\
2\ 5\ 1 \\
\times \quad 1\ 2 \\
\hline
5\ 0\ 2 \\
+\ 2\ 5\ 1\ 0 \\
\hline
3\ 0\ 1\ 2 \\
+\ \ 3 \quad \text{remainder} \\
\hline
3\ 0\ 1\ 5
\end{array}
$$

Example 3   Use long division to evaluate $14{,}539 \div 16$ and show check

To start the long division process since the divisor 16 is larger than the first two leading digits 14 of the dividend 14,439 use the first three leading digits **145** of the dividend **14,5**39. Determine how many times the divisor 16 goes into the leading digits **145** of the dividend? (*may need to do some multiplications on the side*) It goes in 9 times with $9 \times 16 = 144$, so put the 9 in the quotient directly above the 5 in the dividend which is the last digit in the leading digit of the dividend **145**. Then proceed as shown below.

Long Division	**Steps**	**Check**

Long Division

```
            9 0 8  R11
    16 │ 1 4 5 3 9
        −1 4 4 ↓
            1 3
          − 0 0 ↓
            1 3 9
          − 1 2 8
              1 1
```

Steps

$9 \times 16 = 144$

145 $- 144 =$ **1**

Drop digit 3 to form **13**

$0 \times 16 = 0$

13 $- 0 =$ **13**

Drop digit 9 to form **139**

$8 \times 16 = 128$

139 $- 128 =$ **11**

Remainder **11**

Check

```
          4
        9 0 8
      ×   1 6
      5 4 4 8
    + 9 0 8 0
    1 4 5 2 8
      + 1 1    remainder
    1 4 5 3 9
```

To solve application problems requires careful reading to determine which operations are involved in solving the problem. Division is an operation that starts with a total, the dividend, which is broken into a number of common size groups.

Example 4   80 guests are expected to attend a party which is booked in a hall with tables which each have 6 chairs. How many tables are needed to accommodate all the guests?

The dividend is the 80 guests and the common group size is 6 chairs per table. To find the number of groups, the number of tables required, the dividend 80 is divided by the common group size 6. As shown on the right, $80 \div 6 = 13$ R2. Since 13 tables will leave 2 guests without seats, 14 tables are needed to accommodate the 80 guests.

```
        1 3 R2
    6 │ 8 0
      − 6 ↓
        2 0
      − 1 8
          2
```

Example 5   Fred's house has front and back roofs that are both 30 by 15 foot rectangles. If roof shingles are packaged in cases which each can cover 25 square feet, how many cases of shingles are needed to roof Fred's house?

First find the area of the roof. The area of the front roof is the product (30)(15) formed by multiplying the length times the width of the front roof with the area of the entire roof double the area of the front roof. So the area of the roof as shown below is 900 square feet. Now, to find the cases of shingles needed the dividend, the 900 square feet area is divided by common group size of 25 square feet covered by each case of roof shingles. To roof this house 36 cases of shingles are needed.

Area of front roof	$(30)(15) = 450$ square feet	
Area of total roof	$(2)(450) = 900$ square feet	
# of cases of shingles	$900 \div 25 = 36$ cases	

$$
\begin{array}{r}
3\,6 \\
25\,\overline{\smash{\big)}\,9\,0\,0} \\
-\underline{7\,5}\downarrow \\
1\,5\,0 \\
-\underline{1\,5\,0} \\
0
\end{array}
$$

In statistics there are various ways to calculate an average of a list of numbers. The two most common average measurements are the arithmetic **mean** and the **median**. The arithmetic **mean** is derived by first creating a total by adding all the numbers, then dividing that total by how many numbers are listed.

Example 6   The retail price of the five leading tablet computers is $500, $450, $340, $420 and $380. Find the mean price of these tablet computers.

First add the prices of the five tablet computers.

$$500 + 450 + 420 + 380 + 340 = 2090$$

Then divide the total $2090 by the number of computer tablets 5. The mean price of the five leading computer tablets is $418

$$
\begin{array}{r}
4\,1\,8 \\
5\,\overline{\smash{\big)}\,2\,0\,9\,0} \\
-\underline{2\,0}\downarrow \\
0\,9 \\
-\underline{0\,5}\downarrow \\
4\,0 \\
-\underline{4\,0} \\
0
\end{array}
$$

The median is another measurement that calculates the average of a list of numbers. To calculate the **median** average, the numbers are listed in ascending order from smallest to largest and the median average is the number that lies in the middle of the ordered list.

Example 7   The retail price of the five leading tablet computers is $500, $450, $340, $420 and $380. Find the median price of these tablet computers.

List the five values in order from smallest to largest and then cross out two numbers at a time the smallest and largest remaining values on the list until only the middle number 420 is left. The median price of these tablet computers is $420.

$$\overline{340} \quad \overline{380} \quad \mathbf{420} \quad \overline{450} \quad \overline{500}$$

When finding the median of a list that contains an even number of values, there are two numbers left in the middle of the ordered list and the mean of these two middle values is taken to determine the median of the list of values.

Example 8   Find the median age of the following ten community college students: 18, 21, 33, 25, 23, 37, 18, 19, 20, 24

List the ten ages from youngest to oldest. Then cross out two numbers at a time the smallest and largest remaining values on the list until only the two middle numbers 21 and 23 are left. Then calculate the mean of 21 and 23 by adding these two numbers and dividing by two. The median age of these ten students is 22 years.

$$\overline{18} \quad \overline{18} \quad \overline{19} \quad \overline{20} \quad \mathbf{21} \quad \mathbf{23} \quad \overline{24} \quad \overline{25} \quad \overline{33} \quad \overline{37}$$

$$21 + 23 = 44 \qquad \text{and} \qquad 44 \div 2 = 22$$

Exercises 1.7

1-12 Evaluate the following quotients using long division. **Show the checks.**

1. $184 \div 8$

2. $\dfrac{135}{5}$

3. $192 \div 4$

4. $\dfrac{216}{3}$

5. $301 \div 7$

6. $\dfrac{1248}{6}$

7. $2432 \div 8$

8. $\dfrac{396}{11}$

9. $192 \div 12$

10. $\dfrac{3724}{14}$

11. $4832 \div 16$

12. $\dfrac{8115}{15}$

13-21 Evaluate the following quotients using long division. **Show the checks.**

13. $\dfrac{971}{3}$

14. $109 \div 7$

15. $\dfrac{413}{6}$

16. $856 \div 5$

17. $\dfrac{3170}{9}$

18. $341 \div 15$

19. $\dfrac{803}{14}$

20. $907 \div 12$

21. $\dfrac{2258}{11}$

22-35 Solve the following application problems. **Show the calculations.**

22. The price of a fresh fourteen pound turkey is $42. Find the price per pound of this turkey.

23. A gallon of paint covers approximately 400 square feet of drywall. How many gallons cans of paint are needed to cover 2000 square feet of drywall?

24. The nutritional label on a box of cookie lists the serving size as 3 cookies which contain a total of 160 calories. Carlos binges and eats 12 cookies. How many servings of cookies and how many calories did Carlos consume?

25. It cost Alejandro $48 to take his family of four to the movies including drinks and snacks. What is the average cost per person to go to the movies including drinks and snacks?

26.   Sarah has as a goal to save $1800 over the next year. How much does she need to save per month so that at the end of the year she has saved $1800?

27.   A class with 39 students is divided into groups to work on project with 5 students in each group. How many groups are created?

28.   Naomi is building a bookcase that holds approximately 25 books on each shelf. If Naomi has 93 books she wants to store on a bookshelf, how many shelves are needed?

29.   Simone prepares 2 gallons of lemonade for her daughter's party. There are 128 fluid ounces in one gallon. If a plastic children's cup holds 6 fluid ounces, how many cups of lemonade can be filled?

30.   An instructor estimates that 60 pages of paper for handouts/tests will be distributed to each student in the class. The class size is 45 students and each ream of paper contains 500 pages.

30A.   How many pages of paper will the instructor need for this class?
30B.   How many reams of paper will the instructor need for this class?

31.   The nutritional label of saltine crackers list serving size as 5 crackers which contain 150 milligrams of salt. A sleeve of saltine crackers contains approximately 40 crackers.

31A.   How many milligrams of salt are in one cracker?
31B.   How many servings are in a sleeve of crackers?
31C.   How many milligrams of salt are there in a sleeve of crackers?

32.   Find the mean and median of the following ages: 25, 31, 19, 22 and 23

33.   Find the mean and median of the following house prices:

         $200,000 $180,000 $230,000 $170,000

34.   Find the median of following weights in pounds listed below of the five starting offensive lineman of a local professional football team.

                306   329   318   330   323

35.   Find the median height in feet and inches listed below of the eleven players on a professional women's basketball team

       6'4"   6'2"   6'0"   6'8"   5'10"   6'0"   6'4"   5'7"   6'0"   6'1"   6'5"