Section 2.2 Introduction to fractions

In this section, fractions are introduced to represent parts of whole numbers. A fraction breaks a whole number into equal size parts and selects some of these equal size parts. Fractions are written with a bar separating the **numerator** located above the bar which indicates the equal size parts selected and the **denominator** located below the bar which indicates the equal size parts that each of the whole numbers is divided or broken into. Since a whole cannot be divided or broken up into 0 parts, for a fraction to be defined the denominator cannot be zero. As for fraction notation in this textbook, generally when fractions are included in a calculation they are written with a bar and when fractions are included in a sentence the forward slash notation is used.

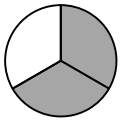
$$\frac{\text{numerator}}{\text{denominator}} \quad \text{or} \quad \text{numerator/denominator}$$

The fraction three fifths written 3/5 has a numerator 3 and denominator 5. This fraction is illustrated below with a rectangle representing the whole number one which is divided into 5 equal size parts with 3 of these parts shaded to indicate that they are selected. Note that the first three equal size parts are shaded, but to represent the fraction 3/5 any three of the five equal parts can be shaded.

$\frac{3}{5}$ 

Example 1 Draw a circle figure which represents the fraction two thirds.

The fraction 2/3 with numerator 2 and denominator 3 is represented as a circle which is divided into 3 equal parts of which two are shaded.



Example 2 Identify the fraction represented by rectangle figure below.

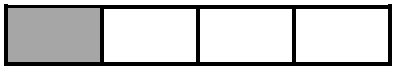
This rectangle diagram represents the whole number one divided into 7 equal parts of which 4 are shaded which is the fraction 4/7 which is read as four sevenths.

A **proper fraction** has a numerator which is smaller than the denominator. Proper fractions are smaller than the whole number one.

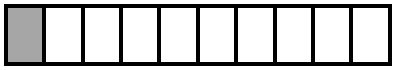
In the United States, the monetary coins include pennies, nickels, dimes, quarters, and half dollars. These coins represent fractional parts of a dollar with 2 half dollars, 4 quarters, 10 dimes, 20 nickels, and 100 pennies all equal to a dollar. Below these coins are written as proper fractions in terms of one dollar (whole unit).

1 half dollar = one half of a dollar = $\$\frac{1}{2}$

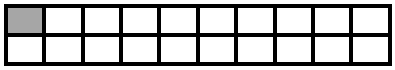
1 quarter = one fourth of a dollar = $\$\frac{1}{4}$



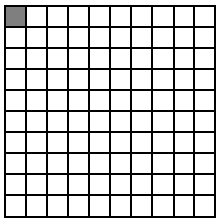
1 dime = one tenth of a dollar = $\$\frac{1}{10}$



1 nickel = one twentieth of a dollar = $\$\frac{1}{20}$



1 penny = one hundredth of a dollar = $\$\frac{1}{100}$



Example 3 Write 3 quarters as a fractional part of a dollar

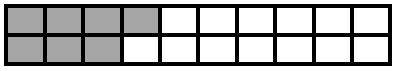
Each quarter represents one fourth of a dollar, so 3 quarters are three fourths of a dollar. The proper fraction 3/4 is shown below as a rectangle which represent one dollar divided into 4 equal parts with 3 parts shaded.

$\$\frac{3}{4}$

Example 4 Write 7 nickels as a fractional part of a dollar

Each nickel represents one twentieth of a dollar, so 7 nickels are seven twentieth of a dollar. The proper fraction 7/20 is shown below as a rectangle which represent one dollar divided into 20 equal parts with 7 parts shaded.

$\$\frac{7}{20}$



An **improper fraction** has a numerator which is larger than or equal to the denominator. Improper fractions are larger than or equal to whole number one.

Below the improper fraction 9/5 with numerator 9 and denominator 5 is illustrated with each rectangle representing the whole number one which is divided into 5 equal size parts with 9 of these parts shaded to indicate that they are selected. Note since the improper fraction 9/5 is greater than one, two rectangles are drawn.

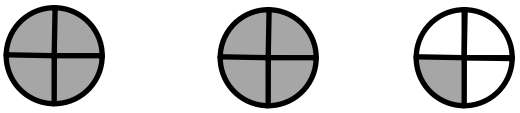
$\frac{9}{5}$ 

Example 5 Identify the following fractions as either proper or improper fractions.
4/7 5/3 9/11 8/8 7/2

$\frac{4}{7}$ and $\frac{9}{11}$ are proper fractions $\frac{5}{3}$, $\frac{8}{8}$ and $\frac{7}{2}$ are improper fractions

Example 6 Draw a circle figure which represents the fraction nine fourths.

The improper fraction 9/4 with numerator 9 and denominator 4 is illustrated with each circle divided into 4 equal size parts with 9 of these parts shaded to indicate that they are selected. When each circle is broken into four equal parts, one full circle contains 4 parts and two full circles contain 8 parts. To illustrate 9/4 two full circles are shaded along with one fourth of a third circle.



Example 7 Identify the improper fraction represented by rectangles below.

Each rectangle is divided into 8 equal parts of which 11 are shaded which is the improper fraction 11/8 which is read as eleven eighths. A common error is to identify the above diagram incorrectly as 11/16 since the two rectangles are broken into a total of 16 equal parts. The correct answer is 11/8, since the denominator indicates how many equal parts one rectangle (one whole unit) is divided into.

A mixed number consists of a whole number combined with a proper fraction.

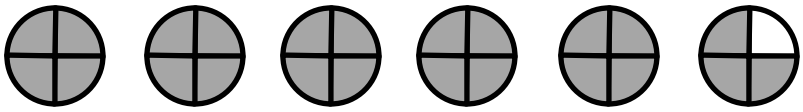
Fractions which are larger than the whole number one can be written as improper fractions or mixed numbers. An advantage of the mixed number format is that since the whole number part is visible the relative size of the number can be immediately determined. When writing mixed numbers using forward slash notation a space is inserted between the whole number and proper fraction. The mixed number 1 4/5 read as one and four fifths is illustrated below as a rectangle (one whole unit) that is divided into 5 equal parts that are all shaded along with a second rectangle that is also divided into 5 equal parts and has 4 parts shaded.

$1\frac{4}{5}$ 

Example 8 Describe in sentence form a rectangle diagram which represents the mixed number three and five sevenths.

To represent 3 5/7 draw four rectangle each of which is divided into 7 equal parts. Shade all the parts in the first three rectangles and in the fourth rectangle shade 5 of the 7 equal parts.

Example 9 Identify the fraction drawn in the circle figure below. Write the fraction in both improper fraction and mixed number formats.



Each circle is divided into 4 equal parts of which 23 are shaded which is the improper fraction 23/4 which is written in words as twenty three fourths.

Since five full circles are shaded the whole number part of the mixed number is 5. The last circle is divided into 4 parts with 3 shaded which is the proper fraction 3/4. The resulting mixed number 5 3/4 is read as five and three fourths.

$$\frac{23}{4} = 5\frac{3}{4}$$

The improper fraction 23/4 and the mixed number 5 3/4 are equal to each since they are both derived from the same circle figure.

To convert an improper fraction into a mixed number

Divide the divisor (denominator) of the improper fraction into the dividend (numerator) using the long division process when necessary. Write the quotient as a mixed number by writing the remainder as the numerator of the proper fraction.

For an improper fraction whose numerator is larger than its denominator the resulting quotient determines how many whole units (rectangles or circles) are fully shaded with the remainder indicating how many parts of the last whole unit (rectangle or circle) are shaded.

Example 10 Write the improper fraction 9/2 as a mixed number.

Dividing 9 by 2 results in the quotient 4 with remainder 1, which can be written as the mixed number 4 1/2 read as four and one half. This quotient can be visualized as four whole units that are each divided into 2 equal parts which are fully shaded as well as a fifth unit that has 1 of the 2 equal parts shaded.

$$\begin{array}{r} 4\ \text{R}1 \\ 2\overline{)\,9} \\ -\,\underline{8} \\ 1 \end{array} \qquad \frac{9}{2} = 4\frac{1}{2}$$

Example 11 Write the improper fraction 17/6 as a mixed number.

Dividing 17 by 6 results in the quotient 2 with remainder 5, which can be written as the mixed number 2 5/6 read as two and five sixths. This quotient can be visualized as two whole units that are each divided into 6 equal parts which are fully shaded as well as a third unit that has 5 of the 6 equal parts shaded.

$$\begin{array}{r} 2\ \text{R}\mathbf{5} \\ 6\overline{)17} \\ -\underline{12} \\ 5 \end{array} \qquad \frac{17}{6} = 2\frac{5}{6}$$

To convert from an improper fraction to a mixed number involves the operation of division. So the reverse of this process to convert from a mixed number to an improper fraction should involve the inverse operation of division which is multiplication. Below the long division process to divide 7 by 3 is shown along with the check process. To check quotients involving remainders show that the product of the quotient and the divisor when added to the remainder equals the dividend.

Improper Fraction	**Long Division**	**Check**	**Mixed Number**
$\frac{7}{3}$	$3\overline{)\,7}$ 2 R**1** $-\underline{6}$ **1**	$2\times3 + \mathbf{1}$ $= 6 + \mathbf{1}$ $= 7$	$2\frac{\mathbf{1}}{3}$

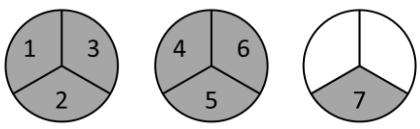
To convert from a mixed number to an improper fraction this check process of calculating the product of the quotient and the divisor and then adding the remainder is used to determine the numerator of the improper fraction.

Example 12 Write the mixed number 2 1/3 as an improper fraction.

For the mixed number 2 1/3, multiply the whole number 2 times the denominator 3 and then add the numerator 1 to obtain 7 as shown below. Thus, 7 is the numerator of the improper fraction 7/3 which is equivalent to the mixed number 2 1/3.

$$2\frac{1}{3} = \frac{(2\cdot3+1)}{3} = \frac{(6+1)}{3} = \frac{7}{3}$$

To illustrate this process the mixed number 2 1/3 is shown below as three circles each of which is divided into three equal parts with all the parts shaded in the first two rectangles and in the third rectangle one of the three equal parts shaded.



To find the numerator of the equivalent improper fraction count the total shaded parts in two circles with all three equal part shaded plus the one shaded part in the third circle which is modeled by 2(3) + 1 which is the same calculation used to check the quotient of 7 and 3.

To convert a mixed number into an improper fraction

Find the product of the whole number part of the mixed number times the denominator and then add the numerator to this product to determine the numerator of the improper fraction. Keep the same denominator used in the original mixed number.

$$A\frac{b}{c} = \frac{(A \cdot c + b)}{c}$$

Example 13 Convert 5 3/7 and 3 2/5 into improper fractions.

To convert the mixed number 5 3/7 into an improper fraction multiply the whole number 5 times the denominator 7 and then add the numerator 3 to obtain 38 which is the numerator of the improper fraction 38/7.

$$5\frac{3}{7} = \frac{(5 \cdot 7 + 3)}{7} = \frac{(35 + 3)}{7} = \frac{38}{7}$$

To convert the mixed number 3 2/5 into an improper fraction multiply the whole number 3 times the denominator 5 and then add the numerator 2 to obtain 17 which is the numerator of the improper fraction 17/5

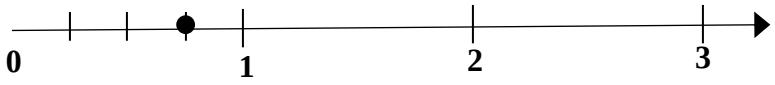
$$3\frac{2}{5} = \frac{(3 \cdot 5 + 2)}{5} = \frac{(15 + 2)}{5} = \frac{17}{5}$$

In the problems in this section when converting mixed numbers to improper fractions and vice versa the steps performed have been shown. But in later sections when converting between these formats when possible the steps will be done mentally without writing out the steps.

A fraction divides whole numbers into equal size parts and selects some of these equal size parts. Fractions are plotted on a number line the same way they are illustrated using rectangle figures by dividing the length between whole numbers on a number line into equal size segments as indicated by the denominator.

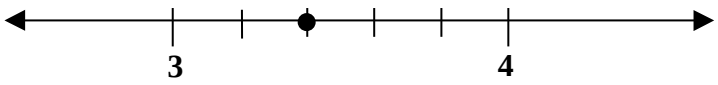
Example 14 Plot the following mixed numbers on a number line: $\frac{3}{4}$ $2\frac{1}{3}$ $3\frac{2}{5}$

3/4 is located between the whole numbers 0 and 1. To plot partition the unit interval connecting the whole numbers 0 and 1 into four equal segments and go 3 segments to the right of the whole number zero as shown below.



2 1/3 is located between the whole numbers 2 and 3. To plot partition the unit interval connecting the whole 2 and 3 into three equal segments and go 1 segment to the right of the whole number two as shown below.

3 2/5 is located between the whole numbers 3 and 4. To plot partition the unit interval connecting the whole numbers 3 and 4 into 5 equal segments and go 2 segments to the left of the whole number 3 as shown below.



Example 15 Describe in sentence form how to plot $5\frac{8}{17}$ on a number line.

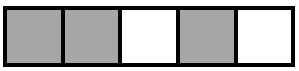
5 8/17 is located between the whole numbers of 5 and 6. To plot break the unit interval between 5 and 6 into 17 equal segments and go 8 of these segments to the right of the whole number 5.

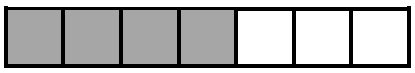
Exercises 2.2

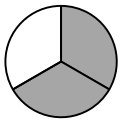
1-6 Write the following as fractions.

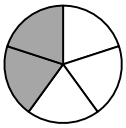
1. Three eights
2. Five ninths
3. Seven thirds
4. Eleven halves
5. Five and two thirds
6. Six and two fifths

7-10 Find the proper fractions represented by the following shaded figures.

7. 

8. 

9. 

10. 

11. Draw a rectangular figure to represent the fraction 5/6
12. Draw a circular figure to represent the fraction 1/2
13. Draw a circular figure representing 3 quarters as a fractional part of a dollar.
14. Draw a rectangular figure representing 7 dimes as a fractional part of a dollar.
15. In sentence form describe a proper fraction.
16. In sentence form describe an improper fraction.

17-20 Identify all the proper and improper fractions in the following lists.

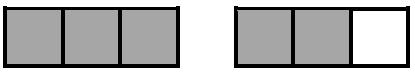
17. 2/3 4/9 8/5 9/2 5/5

18. 11/7 23/25 19/19 17/4

19. $\frac{5}{6}$ $\frac{7}{3}$ $\frac{9}{10}$ $\frac{3}{4}$ $\frac{15}{15}$

20. $\frac{9}{5}$ $\frac{8}{3}$ $\frac{10}{10}$ $\frac{5}{4}$ $\frac{3}{2}$

21-24 Write the fractions represented by the following shaded figures in both improper fraction and mixed number formats.

21. 

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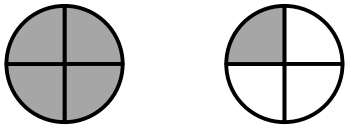
23. 

24. 

25. Draw a rectangular figure to represent the fraction 2 4/5

26. Draw a circular figure to represent the fraction 1 1/2

27. Describe in sentence form a rectangle diagram which represents 5 3/8

28. Describe in sentence form a rectangle diagram which represents 3 1/6

29-37 Write the following improper fractions as mixed numbers.

29. $\frac{9}{2}$ 30. $\frac{11}{3}$ 31. $\frac{7}{5}$

32. $\frac{15}{7}$ 33. $\frac{13}{4}$ 34. $\frac{18}{5}$

35. $\frac{47}{8}$ 36. $\frac{53}{10}$ 37. $\frac{103}{9}$

38-46 Write the following mixed numbers as improper fractions.

38. $2\frac{1}{3}$ 39. $3\frac{1}{2}$ 40. $1\frac{5}{7}$

41. $1\frac{8}{9}$ 42. $4\frac{3}{5}$ 43. $5\frac{1}{6}$

44. $9\frac{2}{7}$ 45. $12\frac{2}{3}$ 46. $13\frac{7}{10}$

47-52 Plot the following fractions on a number line.

47. $\frac{1}{4}$ 48. $\frac{2}{3}$ 49. $1\frac{4}{5}$

50. $2\frac{1}{2}$ 51. $2\frac{3}{4}$ 52. $2\frac{3}{5}$

53-55 Describe in sentences how to plot the following fractions on a number line

53. $8\frac{3}{4}$ 54. $5\frac{4}{9}$ 55. $6\frac{7}{11}$