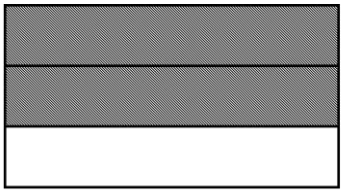
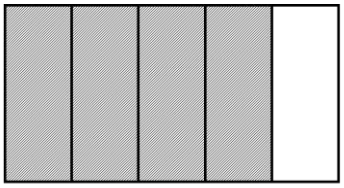
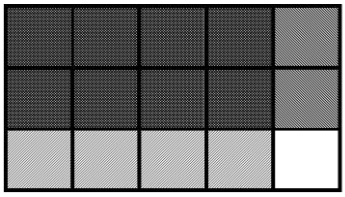
Section 2.4 Multiplying fractions

Some of the keywords that indicate multiplication are product, multiplied by, and times. For fractions another key word that indicates multiplication is "of". To illustrate the multiplication of fractions appropriate parts of the following rectangles are shaded below to calculate 2/3 of 4/5. The fraction 2/3 is represented by breaking the rectangle into three equal horizontal parts with two of these parts shaded and the fraction 4/5 is represented by dividing another rectangle into five equal vertical parts with four of these parts shaded.

$\frac{2}{3}$ 

$\frac{4}{5}$ 

Now to calculate 2/3 of 4/5 these two figures are combined as shown below. First 4/5 is drawn as before by dividing the rectangle into five equal vertical parts with four of these parts shaded. Then to represent 2/3 of 4/5, that same rectangle is now divided into three equal horizontal parts with two of these parts shaded. This results with a rectangle that is divided into 15 equal parts with 8 parts that are shaded twice. Thus, 2/3 of 4/5 is equal to 8/15.

$\frac{2}{3}$ of $\frac{4}{5}$ 

Notice how 2/3 of 4/5 can be easily calculated by simply multiplying the numerators and the denominators separately as shown below.

$$2/3 \text{ of } 4/5 \;=\; \frac{2}{3}\times\frac{4}{5} \;=\; \frac{(2)(4)}{(3)(5)} \;=\; \frac{8}{15}$$

To multiply fractions multiply their numerators and multiply their denominators separately as shown below.

$$\frac{a}{b}\times\frac{c}{d} \;=\; \frac{(a)(c)}{(b)(d)}$$

Example 1 Evaluate the following: $\frac{3}{4} \times \frac{5}{8}$ 2/5 of 8/11 $\left(\frac{5}{7}\right)^2$

Multiply the numerators 3 and 5 and the denominators 4 and 8 separately.

$$\frac{3}{4} \times \frac{5}{8} = \frac{(3)(5)}{(4)(8)} = \frac{15}{32}$$

Multiply the numerators 2 and 8 and the denominators 5 and 11 separately.

$$\frac{2}{5} \text{ of } \frac{8}{11} = \frac{2}{5} \cdot \frac{8}{11} = \frac{(2)(8)}{(5)(11)} = \frac{16}{55}$$

First write 5/7 squared in expanded form as a product with two factors of 5/7. Multiply the numerators 5 and 5 and the denominators 7 and 7 separately.

$$\left(\frac{5}{7}\right)^2 = \frac{5}{7} \cdot \frac{5}{7} = \frac{(5)(5)}{(7)(7)} = \frac{25}{49}$$

When multiplying fractions that have a numerator and denominator with a common factor, the resulting product will not be in reduced form. To demonstrate this below the product of 6/7 and 5/9 is calculated. Since the numerator 6 and denominator 9 have a common factor of 3, the product 30/63 resulting from multiplying the numerators and denominators is not in simplest terms and can be reduced.

$$\frac{6}{7} \times \frac{5}{9} = \frac{(6)(5)}{(7)(9)} = \frac{30}{63} = \frac{(10)(\cancel{3})}{(21)(\cancel{3})} = \frac{10}{21}$$

To multiply fractions that have a numerator and denominator with a common factor it is more efficient to cancel out any common factors first and then multiply the remaining numerators and denominators. Reducing first then multiplying will result with a product that is in reduced form. Below the product of 6/7 and 5/9 is again calculated but this time the GCF 3 of the numerator 6 and denominator 9 is factored out and canceled before the remaining numerators and denominators are multiplied.

$$\frac{6}{7} \times \frac{5}{9} = \frac{(6)(5)}{(7)(9)} = \frac{(2)(\cancel{3})(5)}{(7)(3)(\cancel{3})} = \frac{10}{21}$$

To multiply fractions

First cancel any common factors shared by a numerator and denominator and then multiply the remaining numerators and denominators separately.

Example 2 Evaluate the following: $\frac{4}{15} \times \frac{10}{13}$ $\frac{8}{21} \cdot \frac{7}{10}$

The numerator 10 and denominator 15 have a common factor of 5. First factor out and cancel the 5 and then multiply the remaining numerators and denominators.

$$\frac{4}{15} \times \frac{10}{13} = \frac{(4)(10)}{(15)(13)} = \frac{(4)(2)(\cancel{5})}{(3)(\cancel{5})(13)} = \frac{8}{39}$$

The numerator 8 and denominator 10 have a common factor of 2 and the numerator 7 and denominator 21 have a common factor of 7. First factor out and cancel the 2 and 7 and then multiply the remaining numerators and denominators.

$$\frac{8}{21} \cdot \frac{7}{10} = \frac{(8)(7)}{(21)(10)} = \frac{(4)(\cancel{2})(\cancel{7})}{(3)(\cancel{7})(5)(\cancel{2})} = \frac{4}{15}$$

The cancellation method to reduce a fraction is a shortcut notation in which both the numerator and denominator are both divided by a common divisor with the resulting quotients written above the crossed out numerator and below the crossed out denominator. This same cancellation shortcut is used to multiply fractions that have a numerator and denominator with a common factor. The same fractions in the previous problem are multiplied below using **the cancellation method**.

Example 3 Evaluate the following: $\frac{4}{15} \times \frac{10}{13}$ $\frac{8}{21} \cdot \frac{7}{10}$

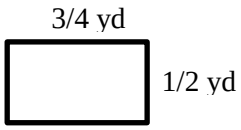
The numerator 10 and denominator 15 have a common factor of 5, so divide both the numerator 10 and denominator 15 by their greatest common factor 5. Now multiply the remaining numerators and denominators separately.

$$\frac{4}{15} \times \frac{10}{13} = \frac{4}{\cancel{15}_{3}} \times \frac{\cancel{10}^{2}}{13} = \frac{8}{39}$$

The numerator 8 and denominator 10 have a common factor of 2, so divide both the numerator 8 and denominator 10 by their GCF 2. The numerator 7 and denominator 21 have a common factor of 7, so divide both the numerator 7 and denominator 21 by their GCF 7. Now multiply the remaining numerators and denominators.

$$\frac{8}{21} \cdot \frac{7}{10} = \frac{\cancel{8}^{4}}{\cancel{21}_{3}} \cdot \frac{\cancel{7}^{1}}{\cancel{10}_{5}} = \frac{4}{15}$$

Example 4 Find the area of the rectangle drawn below.



The area of the rectangle 3/8 square yards is calculated below by multiplying the length times the width of this rectangle.

$$\left(\frac{3}{4}\text{ yd}\right)\left(\frac{1}{2}\text{ yd}\right) = \frac{(3)(1)}{(4)(2)}\text{ yd}^2 = \frac{3}{8}\text{ yd}^2 \text{ or } \frac{3}{8}\text{ sq yd}$$

The multiplication of fractions is easily extended to a product with three or more fractions by simply multiply all the numerators and denominators separately.

Example 5 Evaluate the following: $\frac{2}{5}\cdot\frac{7}{9}\cdot\frac{3}{4}$ $\left(\frac{2}{3}\right)^3$ $\left(\frac{7}{32}\right)\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)$

The numerator 2 and denominator 4 have a common factor of 2, so divide both the numerator 2 and denominator 4 by their GCF 2. The numerator 3 and denominator 9 have a common factor of 3, so divide both the numerator 3 and denominator 9 by their GCF 3. Now multiply the remaining numerators and denominators separately.

$$\frac{2}{5}\cdot\frac{7}{9}\cdot\frac{3}{4} = \frac{\overset{1}{\cancel{2}}}{5}\cdot\frac{7}{\underset{3}{\cancel{9}}}\cdot\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{7}{30}$$

First write 2/3 cubed in expanded form as a product with three factors of 2/3. Since the numerators and denominators do not have a common factor beside one simply multiply the numerators and the denominators separately.

$$\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{(2)(2)(2)}{(3)(3)(3)} = \frac{8}{27}$$

The numerator 4 and denominator 32 have a common factor of 4, so divide both the numerator 4 and denominator 32 by their GCF 4. The numerator 2 and the new denominator 8 have a common denominator of 2, so divide both the numerator 2 and denominator 8 by their GCF 2. Now multiply the remaining numerators and denominators separately.

$$\left(\frac{7}{32}\right)\left(\frac{4}{5}\right)\left(\frac{2}{3}\right) = \frac{7}{\underset{8}{\cancel{32}}}\cdot\frac{\overset{1}{\cancel{4}}}{5}\cdot\frac{\overset{1}{\cancel{2}}}{3} = \frac{7}{60}$$

To multiply fractions involving mixed numbers or whole numbers first convert the mixed numbers and whole numbers into improper fractions then multiply the resulting fractions as previously done.

Example 6 Evaluate the following: $\left(4\frac{2}{3}\right)\left(\frac{5}{7}\right)$ $2\frac{2}{5}\times 1\frac{3}{4}$ $\frac{4}{5}(30)$

First convert the mixed number into an improper fraction. The numerator 14 and denominator 7 have a common factor of 7, so divide both the numerator 14 and denominator 7 by their GCF 7. Then multiply the remaining numerators and denominators separately and write the final answer as a mixed number.

$$\left(4\frac{2}{3}\right)\left(\frac{5}{7}\right) = \frac{14}{3}\cdot\frac{5}{7} = \frac{\cancel{14}^{2}}{3}\cdot\frac{5}{\cancel{7}_{1}} = \frac{10}{3} = 3\frac{1}{3}$$

First convert both mixed numbers into improper fractions. The numerator 12 and denominator 4 have a common factor of 4, so divide both the numerator 12 and denominator 4 by their GCF 4. Then multiply the remaining numerators and denominators separately and write the final answer as a mixed number.

$$2\frac{2}{5}\times 1\frac{3}{4} = \frac{12}{5}\cdot\frac{7}{4} = \frac{\cancel{12}^{3}}{5}\cdot\frac{7}{\cancel{4}_{1}} = \frac{21}{5} = 4\frac{1}{5}$$

First write 30 as the improper fraction 30/1. The numerator 30 and denominator 5 have a common factor of 5, so divide both the numerator 30 and denominator 5 by their GCF 5. Then multiply the remaining numerators and denominators separately and write the final answer as a counting number.

$$\frac{4}{5}(30) = \frac{4}{5}\cdot\frac{30}{1} = \frac{4}{\cancel{5}_{1}}\cdot\frac{\cancel{30}^{6}}{1} = \frac{24}{1} = 24$$

Example 7 Find the area of a rectangle with length 4 2/3 feet and width 1 1/4 feet

The area of this rectangle 5 5/6 square feet is calculated below by multiplying the length times the width of this rectangle.

$$\left(4\frac{2}{3}\text{ ft}\right)\left(1\frac{1}{4}\text{ ft}\right) = \frac{14}{3}\cdot\frac{5}{4}\text{ ft}^2 = \frac{\cancel{14}^{7}}{3}\cdot\frac{5}{\cancel{4}_{2}}\text{ ft}^2 = \frac{35}{6}\text{ ft}^2 = 5\frac{5}{6}\text{ sq ft}$$

Example 8 A community college has a total of approximately 10,000 students with about one fifth of the total students being fulltime students. How many fulltime students are attending this college? How many non-fulltime students are attending this college?

Fulltime students are 1/5 of the 10,000 total students. The number of fulltime students is modeled by the product (1/5)(10,000). As shown below, this college had approximately 2000 fulltime that semester.

$$\frac{1}{5}(10,000) = \frac{1}{5}\cdot\frac{10,000}{1} = \frac{1}{\cancel{5}_{1}}\cdot\frac{\overset{2000}{\cancel{10,000}}}{1} = \frac{2000}{1} = 2000$$

To find the number of non-fulltime students, subtract 2000 the number of fulltime students from 10,000 the total number of students. Since 10,000 – 2000 equals 8000, this college has approximately 8000 non-fulltime students.

Example 9 For a home loan by MortgagesRus the maximum monthly mortgage payment approved is one fourth of the applicant's monthly household income. Find the maximum approved monthly mortgage payment for Josie whose household income is \$4800.

The maximum monthly mortgage payment for Josie is 1/4 of her monthly house household income which is \$4800. Her maximum monthly approved mortgage payment is modeled by the product (1/4)(4800). As shown below, Josie's maximum monthly mortgage payment is \$1200 per month.

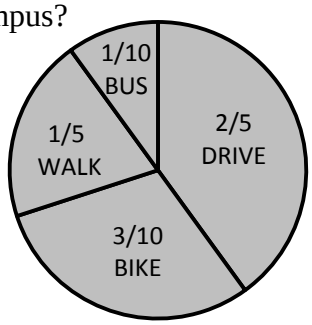
$$\frac{1}{4}(4800) = \frac{1}{4}\cdot\frac{4800}{1} = \frac{1}{\cancel{4}_{1}}\cdot\frac{\overset{1,200}{\cancel{4800}}}{1} = \frac{1200}{1} = 1200$$

Example 10 Lyle completed a five kilometer race in 36 minutes. Jamie who trains daily ran in the same race and finished in 2/3 of the time that Lyle did. What was Jamie's time to complete this race?

Jamie's time is 2/3 of Lyle's time which is 36 minute. Her time is modeled by the product (2/3)(36). As shown below, Jamie completed the race in 24 minutes.

$$\frac{2}{3}(36) = \frac{2}{3}\cdot\frac{36}{1} = \frac{2}{\cancel{3}_{1}}\cdot\frac{\overset{12}{\cancel{36}}}{1} = \frac{24}{1} = 24$$

Example 11 The following pie chart gives the fraction of students at a state university who bike, walk, bus, or drive to campus. If approximately 12,000 students attend this university, estimate how many students drive to campus? How many bike to campus?



Two fifths of the 12,000 students drive to campus. So as shown below approximately 4800 students drive to campus.

$$\frac{2}{5}(12{,}000) = \frac{2}{5}\cdot\frac{12{,}000}{1} = \frac{2}{\cancel{5}_{1}}\cdot\frac{\cancel{12{,}000}^{2400}}{1} = \frac{4800}{1} = 4800$$

Three tenths of the 12,000 students bike to campus. So as shown below approximately 3600 students bike to campus

$$\frac{3}{10}(12{,}000) = \frac{3}{10}\cdot\frac{12{,}000}{1} = \frac{3}{\cancel{10}_{1}}\cdot\frac{\cancel{12{,}000}^{1200}}{1} = \frac{3600}{1} = 3600$$

Example 12 Below is a recipe for one loaf of bread. How much flour is needed to make two loaves? How much milk is needed to make three loaves?

3 2/3 cups flour
1 1/2 teaspoons yeast
2 teaspoons sugar
2 teaspoons oil
1 1/3 cups milk
1 1/2 teaspoons salt

To find the flour for two loaves, double the 3 2/3 cups of flour. As shown below, 7 1/3 cups of flour are needed to make two loaves of this bread.

$$(2)\left(3\frac{2}{3}\right) = \frac{2}{1}\cdot\frac{11}{3} = \frac{22}{3} = 7\frac{1}{3}$$

To find the milk for three loaves, triple the 1 1/3 cups of milk. As shown below, 4 cups of milk are needed to make three loaves of this bread.

$$(3)\left(1\frac{1}{3}\right) = \frac{3}{1}\cdot\frac{4}{3} = \frac{\cancel{3}^{1}}{1}\cdot\frac{4}{\cancel{3}_{1}} = \frac{4}{1} = 4$$

Exercises 2.4

1-33 Evaluate the following and write the final answers in reduced form.

1. $\frac{2}{5} \times \frac{7}{9}$
2. $\frac{5}{6}$ of $\frac{1}{7}$
3. $\left(\frac{4}{5}\right)^2$
4. $\left(\frac{8}{9}\right)^2$
5. $\frac{6}{7} \cdot \frac{5}{9}$
6. $\left(\frac{3}{8}\right)\left(\frac{4}{7}\right)$
7. $\frac{1}{2}$ of $\frac{4}{5}$
8. $\frac{7}{15} \cdot \frac{1}{14}$
9. $\frac{8}{15} \times \frac{5}{12}$
10. $\frac{4}{9} \cdot \frac{3}{8}$
11. $\frac{2}{3}$ of $\frac{9}{10}$
12. $\frac{5}{8} \times \frac{6}{15}$
13. $\left(\frac{25}{14}\right)\left(\frac{21}{30}\right)$
14. $\frac{1}{3} \times \frac{9}{16}$
15. $\frac{1}{3} \cdot \frac{5}{6} \cdot \frac{2}{7}$
16. $\left(\frac{3}{4}\right)\left(\frac{8}{9}\right)\left(\frac{1}{5}\right)$
17. $\left(\frac{2}{3}\right)^3$
18. $\left(\frac{1}{5}\right)^3$
19. $\frac{3}{10} \times \frac{7}{8} \times \frac{15}{14}$
20. $\left(\frac{4}{9}\right)\left(\frac{1}{8}\right)\left(\frac{3}{5}\right)$
21. $\frac{1}{4}(2400)$
22. One fifth of 1000
23. One eighth of 400
24. $\frac{2}{3}$ of 90
25. $\frac{5}{8}$ of 2400
26. Three fourths of 32
27. Four fifths of 80
28. $2\frac{1}{3} \cdot \frac{5}{14}$
29. $\left(3\frac{3}{4}\right)\left(\frac{2}{5}\right)$
30. $4\frac{1}{2} \times \frac{6}{7}$
31. $1\frac{1}{4} \cdot 2\frac{1}{6}$
32. $\left(4\frac{3}{8}\right)\left(1\frac{1}{10}\right)$
33. $1\frac{2}{3} \times 2\frac{2}{5}$

34-43 Solve the following application problems. Show the calculations.

34. A staircase is being built with each stair having a height of 7 1/2 inches. How high will the staircase rise if it will consist of 12 stairs?

35. On a map, the scale is 200 miles per every inch. How far apart in miles are two cities which are 3 1/4 inches apart on the map?

36. Approximately two thirds of the people surveyed supported a measure. If 600 total people were surveyed, estimate how many supported the measure? How many did not support the measure?

37. In the United States Senate, 3/5 of the senators are needed to break a filibuster. Given that that there are 100 senators, how many senators are needed to break a filibuster?

38. A survey found that approximately one fourth of Americans had their data security breached. If 2000 people were surveyed, estimate how many responded that their data security had been breached?

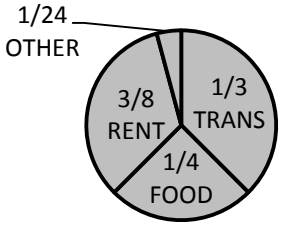
39. A crew of workers can clear the gutters in a condo unit in approximately 2/3 of an hour. Estimate the hours needed by this crew of workers to clear the gutters of a development with 24 condo units.

40. In a survey of 800 Americans, approximately 3/4 of those surveyed believed that climate change is influencing the weather. Estimate the number of people surveyed who believe that climate change is influencing the weather.

41. In 2012, New York City reported that approximately two fifths of their 10 million annual emergency 911 calls were inadvertent so called "butt dials". Estimate how many of New York City's emergency calls were inadvertent?

42. A diet calls for 30 grams of dietary fiber per day. If a serving of black beans has one fourth the recommended daily dietary fiber, how much fiber is there in a serving of black beans?

43. The pie chart below list the fraction of the Nguyen family household budget spend on rent including utilities, food, transportation, and other items. The Nguyen's monthly household budget is $6000. Find the amount budgeted for the three main categories: rent including utilities, transportation, and food.



For #44-48 write final answers with the appropriate units included

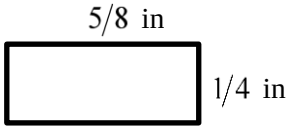
44. Find the area of the rectangle with length 2/3 feet and width 9/10 feet.

45. Find the area of the rectangle with length 3 1/3 yards and width 1 2/5 yards.

46. The display area on a graphing calculator is 2 1/2 inches by 1 1/2 inches. Find the area of the display on this graphing calculator.

47-48. Find the area of the rectangles drawn below.

47.



48.