Section 3.3 Equivalent fractions and least common denominators

To create an equivalent fraction the numerator and denominator of a fraction are multiplied by the same number. Below two equivalent fractions to 3/5 are created by multiplying the original numerator and denominator by two and then by three. Infinite equivalent fractions to 3/5 can be created since multiplying both the original numerator and denominator by the same counting number forms an equivalent fraction. Note, multiplying the numerator and denominator by the same number is simply multiplication by 1 which leaves the value of the original fraction unchanged.

$$\frac{3}{5} = \frac{3(\mathbf{2})}{5(\mathbf{2})} = \frac{6}{10} \qquad \frac{3}{5} = \frac{3(\mathbf{3})}{5(\mathbf{3})} = \frac{9}{15}$$

To add and subtract fractions with unlike denominators instead of randomly creating equivalent fractions, an equivalent fraction with a specific denominator is needed. Suppose that for the fraction 3/5 an equivalent fraction is needed which has a denominator of 30. The detailed steps are shown below to create the fraction 18/30 which is equivalent to 3/5 and has a denominator of 30.

$$\frac{3}{5} = \frac{?}{30} \qquad \frac{3}{5} = \frac{?}{30} \quad \begin{matrix}\textit{Multiply by } 6\\ \\ \textit{Multiply by } 6\end{matrix} \qquad \frac{3}{5} = \frac{3(\mathbf{6})}{5(\mathbf{6})} = \frac{18}{30}$$

To find equivalent fractions with a specific denominator students are not expected to show all the steps shown above. The key observation is that once the multiplication relation between the original and desired denominator is determined that value is also multiplied times the original numerator to find the unknown numerator.

Example 1 Create an equivalent fraction with the indicated denominator.

$\frac{6}{7} = \frac{?}{28}$ (*multiply numerator and denominator by 4)* $\frac{6}{7} = \frac{6(\mathbf{4})}{7(\mathbf{4})} = \frac{24}{28}$

Since 7 times 4 is equal to 28, to find the unknown numerator also multiply the original numerator 6 times 4. The fractions 6/7 and 24/28 are equivalent.

$\frac{1}{4} = \frac{?}{36}$ (*multiply numerator and denominator by 9)* $\frac{1}{4} = \frac{1(\mathbf{9})}{4(\mathbf{9})} = \frac{9}{36}$

Since 4 times 9 is equal to 36, to find the unknown numerator also multiply the original numerator 1 times 9. The fractions 1/4 and 9/36 are equivalent fraction

The **least common denominator (LCD)** of two or more fractions is the least common multiple of the denominators of those fractions.

To add unlike fractions with different denominators, the first step involves finding the least common multiple of the denominators. The techniques introduced earlier in this chapter to find least common multiples are now applied to find the least common multiples of the denominators of the unlike fractions. Most of the least common denominators in this textbook can be quickly found by listing multiples. To find the least common denominator list all the multiples of the largest denominator until a resulting multiple is divisible by all the other denominators. For cases in which listing multiples is not practical, the prime factorization or repeated division method can be used to find the LCD.

Example 2 Find the least common denominator of the following unlike fractions.

7/8 and 3/10 1/4 and 5/6 5/12, 1/6 and 3/5

To find the least common multiple of the denominators 8 and 10, list the multiples of the larger number 10 until a resulting multiple is divisible by the smaller number 8. The LCM of 8 and 10 is 40.

$\frac{7}{8}$ and $\frac{3}{10}$ 10, 20, 30, **40** LCD is 40

To find the least common multiple of the denominators 4 and 6, list the multiples of the larger number 6 until a resulting multiple is divisible by the smaller number 4. The LCM of 4 and 6 is 12.

$\frac{1}{4}$ and $\frac{5}{6}$ 6, **12** LCD is 12

To find the least common multiple of the denominators 5, 6 and 12 list the multiples of the largest number 12 until a resulting multiple is divisible by the both the other numbers 5 and 6. The LCM of 5, 6 and 12 is 60.

$\frac{5}{12}$, $\frac{1}{6}$ and $\frac{3}{5}$ 12, 24, 36, 48, **60** LCD is 60

The next problem combines equivalent fractions with least common denominators. First, the least common denominator of unlike fractions is found and then an equivalent fraction is created for each fraction with the LCD as the denominator. This process is the basis for adding and subtracting unlike fractions.

Example 3 Create equivalent fractions for 3/4 and 2/5 whose denominator is the LCD of these fractions.

First find the LCD by listing the multiples of the larger denominator 5 until a resulting multiple is divisible by the smaller denominator 4. The LCD is 20.

$\frac{3}{4}$ and $\frac{2}{5}$ 5, 10, 15, **20** LCD is 20

Since 4 times 5 is equal to 20, to find the unknown numerator also multiply the original numerator 3 times 5. The fractions 3/4 and 15/20 are equivalent.

$$\frac{3}{4} = \frac{?}{20} \qquad \frac{3}{4} = \frac{3(\mathbf{5})}{4(\mathbf{5})} = \frac{15}{20}$$

Since 5 times 4 is equal to 20, to find the unknown numerator also multiply the original numerator 2 times 4. The fractions 2/5 and 8/20 are equivalent.

$$\frac{2}{5} = \frac{?}{20} \qquad \frac{2}{5} = \frac{2(\mathbf{4})}{5(\mathbf{4})} = \frac{8}{20}$$

Example 4 Create equivalent fractions for 7/9 and 5/6 whose denominator is the LCD of these fractions.

First find the LCD by listing the multiples of the larger denominator 9 until a resulting multiple is divisible by the smaller denominator 6. The LCD is 18.

$\frac{7}{9}$ and $\frac{5}{6}$ 9, **18** LCD is 18

Since 9 times 2 is equal to 18, to find the unknown numerator also multiply the original numerator 7 times 2. The fractions 7/9 and 14/18 are equivalent.

$$\frac{7}{9} = \frac{?}{18} \qquad \frac{7}{9} = \frac{7(\mathbf{2})}{9(\mathbf{2})} = \frac{14}{18}$$

Since 6 times 3 is equal to 18, to find the unknown numerator also multiply the original numerator 5 times 3. The fractions 5/6 and 15/18 are equivalent.

$$\frac{5}{6} = \frac{?}{18} \qquad \frac{5}{6} = \frac{5(\mathbf{3})}{6(\mathbf{3})} = \frac{15}{18}$$

It is easy to compare like fractions visually without doing any steps since the like fraction with the larger numerator is the larger of the two fractions. For unlike fractions creating equivalent fractions with similar denominators serves as a technique to compare the original unlike fractions to determine which is larger.

Example 5 Determine the appropriate symbol < or > to make a true statement

3/4 __ 7/8 4/9 __ 5/12 7/9 __ 5/6

Create equivalent fractions whose denominator is the LCD 8. Since the resulting equivalent fraction 6/8 is less than 7/8, the original fraction 3/4 is less than 7/8.

$$\frac{3(\mathbf{2})}{4(\mathbf{2})} \_\_ \frac{7(\mathbf{1})}{8(\mathbf{1})}$$

$$\frac{6}{8} < \frac{7}{8}$$

$$\frac{3}{4} < \frac{7}{8}$$

Create equivalent fractions whose denominator is the LCD 36. Since the resulting equivalent fraction 16/36 is greater than 15/36, the original fraction 4/9 is greater than 5/12.

$$\frac{4(\mathbf{4})}{9(\mathbf{4})} \_\_ \frac{5(\mathbf{3})}{12(\mathbf{3})}$$

$$\frac{16}{36} > \frac{15}{36}$$

$$\frac{4}{9} > \frac{5}{12}$$

Create equivalent fractions whose denominator is the LCD 18. Since the resulting equivalent fraction 14/18 is less than 15/18, the original fractions 7/9 is less than 5/6.

$$\frac{7(\mathbf{2})}{9(\mathbf{2})} \_\_ \frac{5(\mathbf{3})}{6(\mathbf{3})}$$

$$\frac{14}{18} < \frac{15}{18}$$

$$\frac{7}{9} < \frac{5}{6}$$

The process of creating equivalent fractions with specific denominators also works for mixed numbers. In the next problem, the least common denominator of two mixed numbers is found and then an equivalent fraction for each of these mixed numbers is created with the LCD as the denominator.

Example 6 Create equivalent fractions for 2 5/6 and 7 3/8 whose denominator is the LCD of these mixed numbers.

First find the LCD by listing the multiples of the larger denominator 8 until a resulting multiple is divisible by the smaller denominator 6. The LCD is 24.

$2\frac{5}{6}$ and $7\frac{3}{8}$ 8, 16, **24** LCD is 24

Since 6 times 4 is equal to 24, to find the unknown numerator also multiply the original numerator 5 times 4. The mixed numbers 2 5/6 and 2 20/24 are equivalent.

$$2\frac{5}{6} = 2\frac{?}{24} \qquad 2\frac{5}{6} = 2\frac{5(\mathbf{4})}{6(\mathbf{4})} = 2\frac{20}{24}$$

Since 8 times 3 is equal to 24, to find the unknown numerator also multiply the original numerator 3 times 3. The mixed numbers 7 3/8 and 7 9/24 are equivalent.

$$7\frac{3}{8} = 7\frac{?}{24} \qquad 7\frac{3}{8} = 7\frac{3(\mathbf{3})}{8(\mathbf{3})} = 7\frac{9}{24}$$

To compare mixed numbers with different whole number parts, the mixed number with the larger whole number is larger. To compare mixed numbers with the same whole number parts, create equivalent fractions with similar denominators.

Example 7 Determine the appropriate symbol < or > to make a true statement

$$3\frac{9}{10} \_\_\_ 4\frac{3}{5} \qquad 7\frac{2}{3} \_\_\_ 7\frac{1}{2}$$

Since the whole numbers 3 and 4 of the mixed numbers are different, the mixed number with the larger whole number 4 is the larger of mixed numbers.

$$3\frac{9}{10} < 4\frac{3}{5}$$

Since both mixed numbers have the same whole number part 7, create equivalent fractions whose denominator is the LCD 6. Since the equivalent fraction 7 4/6 is greater than 7 3/6, the original mixed number 7 2/3 is greater than 7 1/2.

$$7\frac{2(\mathbf{2})}{3(\mathbf{2})} \_\_\_ 7\frac{1(\mathbf{3})}{2(\mathbf{3})}$$

$$7\frac{4}{6} > 7\frac{3}{6}$$

$$7\frac{2}{3} > 7\frac{1}{2}$$

Exercises 3.3

1-15. Create an equivalent fraction with the indicated denominator.

1. $\frac{2}{3} = \frac{?}{12}$
2. $\frac{3}{4} = \frac{?}{20}$
3. $\frac{1}{2} = \frac{?}{16}$
4. $\frac{4}{5} = \frac{?}{30}$
5. $\frac{1}{7} = \frac{?}{28}$
6. $\frac{5}{7} = \frac{?}{21}$
7. $\frac{3}{8} = \frac{?}{32}$
8. $\frac{3}{5} = \frac{?}{100}$
9. $\frac{1}{4} = \frac{?}{100}$
10. $5 = \frac{?}{2}$
11. $3 = \frac{?}{4}$
12. $6 = \frac{?}{5}$
13. $7\frac{1}{5} = 7\frac{?}{10}$
14. $4\frac{3}{4} = 4\frac{?}{20}$
15. $5\frac{1}{6} = 5\frac{?}{24}$

16-24 Find the least common denominator of the following unlike fractions.

16. $\frac{1}{4}$ and $\frac{3}{8}$
17. $\frac{2}{5}$ and $\frac{7}{10}$
18. $\frac{3}{4}$ and $\frac{5}{6}$
19. $\frac{5}{6}$ and $\frac{3}{8}$
20. $\frac{1}{6}$ and $\frac{5}{9}$
21. $\frac{3}{10}$ and $\frac{5}{8}$
22. $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$
23. $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$
24. $\frac{3}{4}$, $\frac{2}{5}$ and $\frac{3}{10}$

25-33 Create equivalent fractions whose denominator is the LCD of the fractions.

25. $\frac{1}{2}$ and $\frac{5}{6}$
26. $\frac{3}{4}$ and $\frac{5}{12}$
27. $\frac{5}{6}$ and $\frac{2}{9}$
28. $\frac{1}{6}$ and $\frac{5}{8}$
29. $\frac{3}{5}$ and $\frac{5}{8}$
30. $\frac{3}{4}$ and $\frac{7}{9}$
31. $3\frac{5}{6}$ and $4\frac{1}{9}$
32. $3\frac{5}{8}$ and $1\frac{7}{10}$
33. $1\frac{5}{12}$ and $2\frac{3}{10}$

34-45 Determine the appropriate symbol $<$ or $>$ to make a true statement

34. $\frac{1}{2}$? $\frac{5}{8}$

35. $\frac{4}{5}$? $\frac{7}{10}$

36. $\frac{3}{8}$? $\frac{5}{16}$

37. $\frac{2}{3}$? $\frac{3}{4}$

38. $\frac{7}{9}$? $\frac{5}{6}$

39. $\frac{3}{4}$? $\frac{5}{6}$

40. $3\frac{1}{5}$? $2\frac{7}{10}$

41. $8\frac{1}{2}$? $7\frac{2}{3}$

42. $1\frac{3}{4}$? $2\frac{3}{8}$

43. $3\frac{4}{5}$? $3\frac{9}{10}$

44. $2\frac{5}{9}$? $2\frac{2}{3}$

45. $6\frac{5}{8}$? $6\frac{7}{12}$