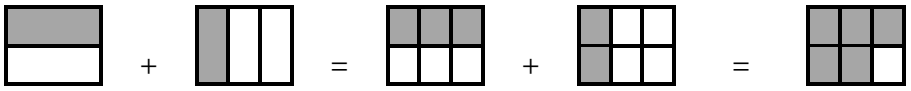
Section 3.4 Adding and subtracting unlike fractions

In this section, fractions with different denominators are added and subtracted by creating equivalent fractions whose denominators are the LCD of the original unlike fractions. Below the process is illustrated with the fraction 1/2 represented as a rectangle broken into 2 equal parts with 1 part shaded and the fraction 1/3 represented as a rectangle broken into 3 equal parts with 1 part shaded. Then equivalent fractions to 1/2 and 1/3 are created by breaking both these rectangles into rectangles with 6 equal parts with appropriate parts shaded. To add the unlike fractions 1/2 and 1/3 their equivalent fractions 3/6 and 2/6 are created which are like fractions which are then added by simply adding their numerators and keeping the common (like) denominator.



1 half + 1 third = 3 **sixths** + 2 **sixths** = 5 **sixths**

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{\mathbf{6}} + \frac{2}{\mathbf{6}} = \frac{5}{\mathbf{6}}$$

To add (or subtract) unlike fractions

1) Find the least common denominator of the fractions.

2) For each fraction create an equivalent fraction with the LCD as the denominator.

3) Add (or subtract) the equivalent fractions which are like fractions by adding (or subtracting) their numerators and keeping the like denominator.

4) If possible reduce the resulting fraction.

This procedure to add and subtract unlike fractions is based on converting unlike fractions into like fractions. To ensure that the resulting equivalent fractions are like fractions the specified denominator is the least common denominator of the original fractions. Once like fractions are created mentally add or subtract the numerators as indicated and keep the like denominator.

To demonstrate this procedure for adding unlike fractions, 1/6 and 3/4 are added below with each step performed separately.

Step 1 Find the LCD 6, **12** LCD is 12

Find the LCD by listing the multiples of the larger denominator 6 until a resulting multiple in this case 12 is divisible by the smaller number 4.

Step 2 Create equivalent fractions whose denominator is the LCD 12

$\frac{1}{6} = \frac{?}{12}$ *(multiply numerator and denominator by 2)* $\frac{1}{6} = \frac{1(\mathbf{2})}{6(\mathbf{2})} = \frac{2}{12}$

$\frac{3}{4} = \frac{?}{12}$ *(multiply numerator and denominator by 3)* $\frac{3}{4} = \frac{3(\mathbf{3})}{4(\mathbf{3})} = \frac{9}{12}$

Step 3 Add the like fractions 2/12 and 9/12 $\frac{2}{12} + \frac{9}{12} = \frac{11}{12}$

The like fractions 2/12 and 9/12 are added mentally by simply adding their numerators 2 and 9 and keeping the common denominator 12.

Step 4 Write 11/12 is in reduced from $\frac{11}{12}$

11/12 is in reduced from since the greatest common factor of 11 and 12 is one.

Instead of performing the four steps separately, the addition and subtraction of unlike fractions is displayed in either a vertical or horizontal format. Below the fractions 1/6 and 3/4 are added again with the steps written in vertical and horizontal formats. Students can choose whichever format is more convenient for them, but most algebra textbooks usually display addition and subtraction of fractions in a horizontal format.

Vertical Format

$$\begin{array}{rcccc} \frac{1}{6} & = & \frac{1(\mathbf{2})}{6(\mathbf{2})} & = & \frac{2}{12} \\ + \frac{3}{4} & = & \frac{3(\mathbf{3})}{4(\mathbf{3})} & = & \frac{9}{12} \\ \hline & & & & \frac{11}{12} \end{array}$$

Horizontal Format

$$\frac{1}{6} + \frac{3}{4} = \frac{1(\mathbf{2})}{6(\mathbf{2})} + \frac{3(\mathbf{3})}{4(\mathbf{3})} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$$

Example 1 Add $\frac{3}{5} + \frac{7}{10}$ (vertical format)

To find the LCD since the larger denominator 10 is divisible by 5 no other multiples need to be listed. Create equivalent fractions whose denominator is the LCD 10 and then add the like fractions. Write the final answer as the mixed number 1 3/10 as shown below.

10 LCD is 10

$$\begin{array}{rrcl} & \frac{3(\mathbf{2})}{5(\mathbf{2})} & = & \frac{6}{10} \\ + & \frac{7(\mathbf{1})}{10(\mathbf{1})} & = & \frac{7}{10} \\ \hline & \frac{13}{10} & = & 1\frac{3}{10} \end{array}$$

Example 2 Add $\frac{1}{6} + \frac{5}{8}$ (vertical format)

To find the LCD list the multiples of the larger denominator 8 until a resulting multiple in this case 24 is divisible by the smaller number 6. Create equivalent fractions whose denominator is the LCD 24 and then add the like fractions. The final answer is 19/24 as shown below.

8, 16, **24** LCD is 24

$$\begin{array}{rrcl} & \frac{1(\mathbf{4})}{6(\mathbf{4})} & = & \frac{4}{24} \\ + & \frac{5(\mathbf{3})}{8(\mathbf{3})} & = & \frac{15}{24} \\ \hline & \frac{19}{24} & & \end{array}$$

Example 3 Subtract $\frac{1}{2} - \frac{3}{10}$ (horizontal format)

To find the LCD since the larger denominator 10 is divisible by 2 no other multiples need to be listed. Create equivalent fractions whose denominator is the LCD 10 and subtract the like fractions. Reduce the final answer to 1/5 as shown below.

10 LCD is 10

$$\frac{1}{2} - \frac{3}{10} = \frac{1(\mathbf{5})}{2(\mathbf{5})} - \frac{3}{10} = \frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{\cancel{2}^{\,1}}{\cancel{10}_{\,5}} = \frac{1}{5}$$

Example 4 Add $\frac{2}{3} + \frac{1}{6} + \frac{2}{9}$ (vertical format)

To find the LCD list the multiples of the larger denominator 9 until a resulting multiple in this case 18 is divisible by both the smaller denominators 3 and 6. Create equivalent fractions whose denominator is the LCD 18 and add the like fractions. Write the final answer as the mixed number 1 1/18 as shown below.

9, **18** LCD is 18

$$\begin{aligned} \frac{2(\mathbf{6})}{3(\mathbf{6})} &= \frac{12}{18} \\ \frac{1(\mathbf{3})}{6(\mathbf{3})} &= \frac{3}{18} \\ + \frac{2(\mathbf{2})}{9(\mathbf{2})} &= \frac{4}{18} \\ \hline \frac{19}{18} &= 1\frac{1}{18} \end{aligned}$$

Example 5 Add $\frac{1}{3} + \frac{3}{4} + \frac{2}{5}$ (horizontal format)

To find the LCD list the multiples of the larger denominator 5 until a resulting multiple in this case 60 is divisible by both the smaller denominators 3 and 4. Create equivalent fractions whose denominator is the LCD 60 and add the like fractions. Write the final answer as a mixed number 1 29/60 as shown below.

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, **60** LCD is 60

$$\frac{1}{3} + \frac{3}{4} + \frac{2}{5} = \frac{1(\mathbf{20})}{3(\mathbf{20})} + \frac{3(\mathbf{15})}{4(\mathbf{15})} + \frac{2(\mathbf{12})}{5(\mathbf{12})} = \frac{20}{60} + \frac{45}{60} + \frac{24}{60} = \frac{89}{60} = 1\frac{29}{60}$$

Example 6 It takes Lisa 5/6 of an hour to complete an assignment working alone. When she works with a partner she can finish the assignment in a 1/2 hour. How much time does Lisa save by working with a partner?

To find the unknown quantity, how much time Lisa saves by working with a partner on an assignment, subtract the time when working with a partner 1/2 hour from the time required when she works alone 5/6 of an hour. As shown below Lisa saves 1/3 of an hour by working with a partner.

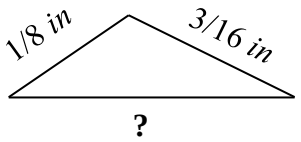
6 LCD is 6

$$\frac{5(\mathbf{1})}{6(\mathbf{1})} = \frac{5}{6}$$

$$-\frac{1(\mathbf{3})}{2(\mathbf{3})} = -\frac{3}{6}$$

$$\frac{2}{6} = \frac{\cancel{2}^{1}}{\cancel{6}_{3}} = \frac{1}{3}$$

Example 7 Find the missing side of the triangle given the perimeter is 9/16 inch.



To find the unknown quantity, the length of the bottom side of the triangle, first add the lengths of the two given sides of the triangle 1/8 and 3/16 inches and then subtract that amount from the length of the given perimeter 9/16 inches as shown below. The length of the bottom side of the triangle is 1/4 inches.

Add the two sides (unlike fractions with LCD 16)

$$\frac{1}{8} + \frac{3}{16} = \frac{1(\mathbf{2})}{8(\mathbf{2})} + \frac{3}{16} = \frac{2}{16} + \frac{3}{16} = \frac{5}{16}$$

Find the difference of the perimeter and the sum of the two sides (like fractions)

$$\frac{9}{16} - \frac{5}{16} = \frac{4}{16} = \frac{\cancel{4}^{1}}{\cancel{16}_{4}} = \frac{1}{4}$$

Exercises 3.4

1-12 Add the following fractions. Write final answers in reduced form.

1. $\frac{1}{2} + \frac{3}{4}$
2. $\frac{2}{5} + \frac{3}{10}$
3. $\frac{5}{6} + \frac{3}{8}$
4. $\frac{1}{9} + \frac{5}{6}$
5. $\frac{3}{16} + \frac{3}{8}$
6. $\frac{3}{4} + \frac{1}{6}$
7. $\frac{2}{3} + \frac{1}{4}$
8. $\frac{3}{10} + \frac{7}{8}$
9. $\frac{7}{12} + \frac{1}{8}$
10. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$
11. $\frac{4}{5} + \frac{1}{6} + \frac{3}{10}$
12. $\frac{2}{3} + \frac{1}{4} + \frac{2}{5}$

13-21 Subtract the following fractions. Write final answers in reduced form.

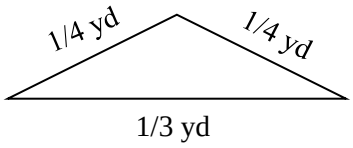
13. $\frac{5}{6} - \frac{2}{3}$
14. $\frac{3}{4} - \frac{1}{2}$
15. $\frac{6}{7} - \frac{3}{5}$
16. $\frac{7}{9} - \frac{2}{3}$
17. $\frac{3}{8} - \frac{1}{12}$
18. $\frac{13}{15} - \frac{7}{10}$
19. $\frac{4}{5} - \frac{3}{4}$
20. $\frac{5}{12} - \frac{1}{9}$
21. $\frac{3}{4} - \frac{7}{16}$

22-27 Evaluate using the order of operations. Write final answers in reduced form.

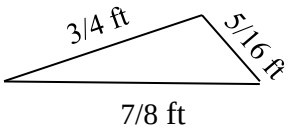
22. $\frac{4}{5} + \frac{3}{10} - \frac{1}{2}$
23. $\frac{4}{5} - \frac{3}{10} + \frac{1}{2}$
24. $\frac{15}{16} - \frac{3}{8} + \frac{1}{4}$
25. $\frac{15}{16} - \left(\frac{3}{8} + \frac{1}{4}\right)$
26. $\frac{2}{3} - \left(\frac{1}{6} + \frac{4}{9}\right)$
27. $\frac{2}{3} - \frac{1}{6} + \frac{4}{9}$

28-29 Find the perimeter of the following triangles.

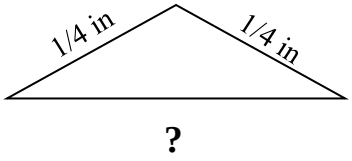
28.



29.



30. Find the missing side of the following triangle whose perimeter is 7/8 inches.



31-35 Solve the following application problems. Show the calculations.

31. Approximately 1/5 of the world population lives in China and 1/20 of the world population live in the United States. What fraction of the world population lives in either China or the United States?

32. A truck filled with sand delivers 2/5 of the total sand at the first stop and 3/8 at the second stop. Which stop had the larger delivery? What fraction of the total sand is delivered during the first two stops?

33. How much larger is a 7/16 inch length screw than a 1/4 inch length screw?

34. A serving of beans has approximately 1/4 of the daily recommended fiber and an apple has about 1/6 of the daily recommended amount. If a serving of beans and one apple are eaten, what fraction of the daily recommended fiber is consumed?

35. The pie chart below list the fraction of the Nguyen family household budget spend on rent including utilities, food, transportation, and other items. Which category is the largest part of the budget? What fraction part of the budget is allocated for both rent including utilities and food?