

Section 4.1 Introduction to decimals

In chapter one, whole numbers are written in standard format using the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 along with a positional base ten system. In chapters two and three, fractions are introduced which break up a whole number into parts. In this section, decimals are introduced as an alternative to fractions. Terminating decimals are fractions whose denominators are restricted to the place value units such as tens, hundreds, thousands, ten-thousands, hundred-thousands, and millions. The decimal system extends the base ten system to the right of the ones place with the decimal point acting as a border separating the whole and decimal (fractional) parts of a number. For whole numbers located on the left side of the decimal point, the place values start at the ones place and each successive larger place value is equal to the place value located to its immediate right multiplied by 10. For decimals located on the right hand side of the decimal point, the place values start at the tenths place and each successive smaller place value is equal the place value to its immediate left divided by 10. This multiplying and dividing of successive place values by 10 results in reciprocal relationships between corresponding place values such as the tens and tenths, hundreds and hundredths, and thousand and thousandths. Notice that since division by one leaves a whole number unchanged; there is no oneths place with the first place value to the right of the decimal point being the tenths place.

$10 \times 10,000 = 100,000$ hundred-thousands	$10 \times 1,000 = 10,000$ ten-thousands	$10 \times 100 = 1,000$ thousands	$10 \times 10 = 100$ hundreds	$10 \times 1 = 10$ tens	1 ones	Decimal point	$1/(10) = 1/10$ tenths	$1/(10 \times 10) = 1/100$ hundredths	$1/(10 \times 100) = 1/1,000$ thousandths	$1/(10 \times 1,000) = 1/10,000$ ten-thousandths	$1/(10 \times 10,000) = 1/100,000$ hundred-thousandths
						.					

In fraction format, proper fractions have numerators smaller than their denominators and are less than one. In decimal format, proper decimals are also smaller than one. When writing proper decimals it is customary to put a zero as the whole number. While writing the 0 whole is not required, when it is not written a reader can easily miss the small decimal point. In the nursing field, writing the 0 whole is required to avoid errors transcription errors by busy medical professionals who may inadvertently miss the small decimal points. In fact, many countries around the world including most countries in South America and Europe use a comma instead of a small dot to indicate the decimal point.

To write a proper decimal in words

Treat the digits to the right of the decimal place as a whole number and write that whole number in words followed by the place value name of the location of the last digit.

In this section the last digit is bolded along with its place value name in the solutions to visualize the process of writing a decimal in words but this practice will be discontinued in later sections.

Example 1 Write the following proper decimals in words and in fraction format.

0.13 0.208 0.0591 0.000027

The decimal 0.13 is read as thirteen **hundredths** since the last digit 3 is in the **hundredths** place.

$$0.13 = \text{thirteen } \mathbf{hundredths} = \frac{13}{100}$$

The decimal 0.208 is read as two hundred eight **thousandths** since the last digit 8 is in the **thousandths** place.

$$0.208 = \text{two hundred eight } \mathbf{thousandths} = \frac{208}{1000}$$

The decimal 0.0591 is read as five hundred ninety one **ten-thousandths** since the last digit 1 is in the **ten-thousandths** place.

$$0.0591 = \text{five hundred ninety one } \mathbf{ten-thousandths} = \frac{591}{10,000}$$

The decimal 0.000027 is read as twenty seven **millionths** since the last digit 7 is in the **millionths** place.

$$0.000027 = \text{twenty seven } \mathbf{millionths} = \frac{27}{1,000,000}$$

Example 2 Write the following in decimal and fraction format.

seven tenths thirty two hundredths four hundred thirty five ten-thousands

To write seven tenths as a decimal to the right of the decimal point place put the number 7 with the last digit 7 is in the tenths place.

$$\text{seven } \mathbf{tenths} = 0.7 = \frac{7}{10}$$

To thirty two hundredths as a decimal to the right of the decimal point place put the number 32 with the last digit 2 is in the hundredths place.

$$\text{thirty two } \mathbf{hundredths} = 0.32 = \frac{32}{100}$$

To write four hundred thirty five ten-thousands as a decimal to the right of the decimal point place put the number 435 with the last digit 5 is in the ten thousandths place and fill in the blank tenths place with a zero digit.

$$\text{four hundred thirty five } \mathbf{ten-thousands} = 0.0435 = \frac{435}{10,000}$$

Example 3 Write the following fractions in words and in decimal format.

$$3/10$$

$$81/100$$

$$9/1000$$

To write the fraction 3/10 as a decimal to the right of the decimal point place put the number 3 with the last digit 3 is in the tenths place.

$$\frac{3}{10} = \text{three } \mathbf{tenths} = 0.3$$

To write the fraction 81/100 as a decimal to the right of the decimal point place put the number 81 with the last digit 1 is in the hundredths place.

$$\frac{81}{100} = \text{eighty one } \mathbf{hundredths} = 0.81$$

To write the fraction 9/1000 as a decimal to the right of the decimal point place put the number 9 with the last digit 9 is in the thousandths place and fill in the blank tenths and hundredth places with zero digits.

$$\frac{9}{1000} = \text{nine } \mathbf{thousandths} = 0.009$$

A mixed number is a whole number combined with a proper fraction. Similarly a mixed decimal is a whole number combined with a proper decimal. In the next few problems, mixed decimals are written in words, decimal format, and fraction format.

To write a mixed decimal in words

Write the whole number to the left of the decimal point followed by the word “and” to separate the whole number and the decimal part. Then write the proper decimal part in words as before.

Example 4 Write the following decimals in words and in fraction format.

7.645

212.03

Write the whole number seven followed by the word “and”. The decimal is read as six hundred forty five thousandths since the last digit 5 is in the thousandths place.

$$7.645 = \text{seven and six hundred forty five } \mathbf{\text{thousandths}} = 7\frac{645}{1000}$$

Write the whole number two hundred twelve followed by the word “and”. The decimal is read as three hundredths since the last digit 3 is in the hundredths place.

$$212.03 = \text{two hundred twelve and three } \mathbf{\text{hundredths}} = 212\frac{3}{100}$$

Example 5 Write the following in decimal and fraction format.

nine and eight tenths

eighteen and nine hundredths

Put the whole number 9 on the left of the decimal point and to the right of the decimal point place put the number 8 with the last digit 8 is in the tenths place.

$$\text{nine and eight } \mathbf{\text{tenths}} = 9.8 = 9\frac{8}{10}$$

Put the whole number 18 on the left of the decimal point and to the right of the decimal point place put the number 9 with the last digit 9 is in the hundredths place and fill in the blank tens place with a zero digit.

$$\text{eighteen and nine } \mathbf{\text{hundredths}} = 18.09 = 18\frac{9}{100}$$

Example 6 Write the following mixed numbers in words and in decimal format.

$$93 \frac{17}{100}$$

$$8 \frac{62}{1000}$$

To the left of the decimal point put the whole number 93 and to the right of the decimal point place put the number 17 with the last digit 7 is in the hundredths place.

$$93 \frac{17}{100} = \text{ninety three and seventeen } \mathbf{hundredths} = 93.17$$

To the left of the decimal point put the whole number 8 and to the right of the decimal point place put the number 62 with the last digit 2 is in the thousandths place filling in the blank tenth place with a zero digit.

$$8 \frac{62}{1000} = \text{eight and sixty two } \mathbf{thousandths} = 8.062$$

When writing a decimal in words the place value name of where the last digit on the right is located is written out. Another way to describe a decimal is to mention how many decimal places the number has. **Decimal places** indicate how many digits are located to the right of the decimal place.

Example 7 Find the number of decimal places in the following decimals.

0.**345** has three decimal places

23.**7** has one decimal place

9.**04** has two decimal places

17 has no decimal places

In earlier chapters, monetary coins are written as fractional parts of a dollar. But, in most cases monetary coins are easier to work with if they are written as decimals. Since one cent is one hundredth of a dollar, a cent can be written in decimal format as \$0.01 while the other coins can be mentally converted into cents then written as decimals with two decimal places.

$$\text{nickel} = 5 \text{ cents} = \$0.05$$

$$\text{quarter} = 25 \text{ cents} = \$0.25$$

$$\text{dime} = 10 \text{ cents} = \$0.10$$

$$\text{half dollar} = 50 \text{ cents} = \$0.50$$

Example 8 Write the following monetary values in words.

\$12.95 Twelve dollars and ninety five cents

\$19 Nineteen dollars

\$1.06 One dollar and six cents

\$0.70 Seventy cents

In chapter one, metric prefixes such as kilo, mega, and giga are introduced to indicate large measured quantities. Metric prefixes also indicate small measured quantities that occur especially in scientific and medical applications. Below some common metric prefixes that indicate small quantities are defined.

The prefix **milli** indicates a thousandth

The prefix **micro** indicates a millionth

The prefix **nano** indicates a billionth

Example 9 Write the following measurements using the appropriate prefix.

0.000045 meters 0.123 grams 0.000000007 meters

0.000045 meters = 45 **millionths** of a meter = 45 **micrometers**

0.123 grams = 123 **thousandths** of a gram = 123 **milligrams**

0.000000007 meters = 7 **billionths** of a meter = 7 **nanometers**

Example 10 Write the following measurements without metric prefixes.

250 milliliters 15 micrograms 2 nanometers

250 **milliliters** = 250 **thousandths** of a liter = 0.250 liters

15 **micrograms** = 15 **millionths** of a gram = 0.000015 grams

2 **nanometers** = 2 **billionths** of a meter = 0.000000002 meters

Exercises 4.1

1-6 Write the following in both fraction and decimal format.

1. Two hundred thirty six thousandths
2. Forty six hundredths
3. Five and thirty eight hundredths
4. Twenty three and six tenths
5. Six and eight thousandths
6. Fifty three and three hundredths

7-15 Write the following decimals in words and in fraction format.

- | | | |
|-----------|--------------|-----------|
| 7. 0.19 | 8. 0.356 | 9. 0.06 |
| 10. 0.003 | 11. 1.7 | 12. 2.35 |
| 13. 5.045 | 14. 0.000018 | 15. 0.503 |

16-24 Write the following fractions in words and in decimal format.

- | | | |
|-------------------------|---------------------------|----------------------------|
| 16. $\frac{83}{100}$ | 17. $\frac{7}{10}$ | 18. $\frac{89}{1000}$ |
| 19. $\frac{9}{100}$ | 20. $4\frac{53}{100}$ | 21. $6\frac{307}{1000}$ |
| 22. $11\frac{93}{1000}$ | 23. $\frac{145}{100,000}$ | 24. $\frac{59}{1,000,000}$ |

25-30 Find the number of decimal places in the following decimals.

- | | | |
|----------|-----------|------------|
| 25. 0.12 | 26. 0.509 | 27. 0.0035 |
| 28. 11.8 | 29. 2.35 | 30. 4.0986 |

31-34 Write the following monetary values as decimals.

- 31. Twenty five dollars and thirty eight cents
- 32. One hundred forty two dollars and nine cents
- 33. Two thousand, three hundred eight dollars and forty cents
- 34. Eight hundred dollars and ninety nine cents

35-40 Write the following monetary values in words.

- | | | |
|-------------|-------------|---------------|
| 35. \$0.83 | 36. \$0.06 | 37. \$1.35 |
| 38. \$12.83 | 39. \$18.09 | 40. \$1056.99 |

41-44 Write the following measurements in words and then substitute the appropriate metric prefix (*see example 9 on page 156*)

- | | |
|--------------------|------------------------|
| 41. 0.017 meters | 42. 0.008 grams |
| 43. 0.000185 grams | 44. 0.000000012 meters |

45-48 Write the following measurements without metric prefixes (*see example 10 on page 157*)

- | | |
|--------------------|-------------------|
| 45. 45 milligrams | 46. 5 milliliters |
| 47. 325 micrograms | 48. 98 nanometers |
49. The daily recommended amount of zinc is 15 milligrams. Write 15 milligrams without the metric prefix.
50. Visible light has a wavelength between 400 and 750 nanometers. Write 750 nanometers without the metric prefix.
51. The length of an Ebola virus is approximately 970 nanometers while the length of an E. coli bacterium is about 3 micrometers. Write both 970 nanometers and 3 micrometers without the metric prefixes.