

Section 4.3 Converting fractions into decimal format

Earlier in this chapter, fractions with denominators that are place numbers such as tens, hundreds, and thousands are converted into decimal format by writing the fraction in words and creating the resulting decimal as shown in the problem below.

Example 1 Write the fraction $17/1000$ as a decimal

The fraction $17/1000$ is read as seventeen thousandths. To the right of the decimal point place put the number 17 with the last digit 7 is in the thousandths place and fill in the blank tenths with a zero digit.

$$\frac{17}{1000} = \text{seventeen thousandths} = 0.017$$

But, how are fractions whose denominators are not place value numbers written as decimals? In this section, fractions are converted into a decimal format by dividing the denominator into the numerator using long division. To obtain a decimal quotient the dividend (numerator) is written inside the long division box with a decimal point and zero digits to the right of the decimal point. To demonstrate this procedure, the fraction $3/4$ is converted into a decimal format by using long division with the dividend (numerator) 3 written with a decimal point followed by zero digits and the decimal points in the dividend and quotient lined up. This results with the fraction $3/4$ written in decimal format as 0.75 as shown below.

$$\begin{array}{r} .75 \\ 4 \overline{) 3.00} \\ \underline{-28} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Line up the decimal places in quotient and dividend

Write whole number 3 as 3.00

7 times 4 equals 28

Drop a zero digit

5 times 4 equals 20

0 remainder terminating decimal

$$\frac{3}{4} = 0.75$$

$$\text{Check: } 0.75 = \frac{75}{100} = \frac{(3)(\cancel{25})}{(4)(\cancel{25})} = \frac{3}{4}$$

Example 2 Write the fraction $2/5$ as a decimal.

Write $2/5$ in long division format as 2 divided by 5 with the dividend 2 written with a decimal point followed by zero digits and the decimal places in the quotient and dividend lined up. This results with the fraction $2/5$ equal to the decimal 0.4 as shown below.

$$\begin{array}{r} .4 \\ 5 \overline{) 2.0} \\ \underline{-2 \ 0} \\ 0 \end{array}$$

Line up the decimal places in quotient and dividend

Write the dividend 2 as 2.0

4 times 5 equals 20

0 remainder terminating decimal

$$\frac{2}{5} = 0.4$$

To convert mixed numbers into decimals simply write the proper fraction part of the mixed number as a decimal using long division and then attach the whole number part of the mixed number to that decimal.

Example 3 Write the mixed number $2 \frac{7}{8}$ as a decimal.

For the mixed number $2 \frac{7}{8}$ write the proper fraction $7/8$ in long division format as 7 divided by 8 with the dividend 7 written with a decimal point followed by zero digits and the decimal places in the quotient and dividend lined up. This results with the mixed number $2 \frac{7}{8}$ equal to the decimal 2.875 as shown below.

$$\begin{array}{r} .875 \\ 8 \overline{) 7.000} \\ \underline{-6 \ 4} \downarrow \\ \mathbf{6 \ 0} \\ \underline{-5 \ 6} \downarrow \\ \mathbf{4 \ 0} \\ \underline{-4 \ 0} \\ 0 \end{array}$$

Line up the decimal places in quotient and dividend

Write the dividend 7 as 7.000

8 times 8 equals 64

Drop a zero digit

7 times 8 equals 56

Drop a zero digit

5 times 8 equals 40

0 remainder terminating decimal

Attach the whole number 2

$$2\frac{7}{8} = 2.875$$

The fractions $3/4$, $2/5$ and $7/8$ when divided using long division have a zero remainder and are terminating decimals. But this is not usually the case. Fractions in reduced form with denominators that have prime factors besides 2 or 5 will result in repeating decimals. Repeating decimals repeat either a single or multi digit pattern forever which is indicated by writing a bar over the repeated pattern. When using the repeated decimals in calculations they are rounded to a specified place value depending on how much precision is required in that particular problem.

For fractions in reduced form whose denominators have prime factors that are limited to 2 and 5 when divided using long division a zero remainder will eventually occur resulting in a **terminating decimal**.

For fractions in reduced form whose denominators have a prime factor besides 2 or 5 when divided using long division a repeating digit pattern will eventually occur. This results in a **repeating decimal** that is indicated with a bar over the repeating digit pattern.

Example 4 Write $2/3$ as a decimal and round to the hundredths place.

Write $2/3$ in long division format as 2 divided by 3 with the numerator 3 written with a decimal point followed by zero digits and the decimal places in the quotient and dividend lined up as shown below.

$$\begin{array}{r}
 .666 \\
 3 \overline{) 2.000} \\
 \underline{-18} \\
 20 \\
 \underline{-18} \\
 20 \\
 \underline{-18} \\
 1
 \end{array}$$

Line up the decimal places in quotient and dividend

Write the dividend 2 as 2.000

6 times 3 equals 18

Drop a zero digit

6 times 3 equals 18

Drop a zero digit

6 times 3 equals 18

Repeating digit 6

Round to the nearest hundredth

$$\frac{2}{3} = 0.\overline{6} \approx 0.67$$

Example 5 Write $9/11$ as a decimal and round to the hundredths place.

Write $9/11$ in long division format as 9 divided by 11 with the dividend 9 written with a decimal point followed by zero digits and the decimal places in the quotient and dividend lined up as shown below.

$$\begin{array}{r}
 .8181 \\
 11 \overline{) 9.0000} \\
 \underline{-88} \\
 20 \\
 \underline{-11} \\
 90 \\
 \underline{-88} \\
 20 \\
 \underline{-11} \\
 9
 \end{array}$$

Line up the decimal places in quotient and dividend

Write the dividend 9 as 9.0000

8 times 11 equals 88

Drop a zero digit

1 times 11 equals 11

Drop a zero digit

8 times 11 equals 88

Drop a zero digit

1 times 11 equals 11

Repeating digits 81

Round to the nearest hundredth

$$\frac{9}{11} = 0.\overline{81} \approx 0.82$$

Example 6 Write $5 \frac{1}{6}$ as a decimal and round to two decimal places.

For the mixed number $5 \frac{1}{6}$ write the proper fraction part $1/6$ in long division format as 1 divided by 6 with the dividend 1 written with a decimal point followed by zero digits and the decimal places in the quotient and dividend lined up as shown below. Then attach the whole number 5 to the resulting decimal.

$$\begin{array}{r}
 .166 \\
 6 \overline{) 1.000} \\
 \underline{-6} \\
 40 \\
 \underline{-36} \\
 40 \\
 \underline{-36} \\
 4
 \end{array}$$

Line up the decimal places in quotient and dividend

Write the dividend 1 as 1.000

1 times 6 equals 6

Drop a zero digit

6 times 6 equals 36

Drop a zero digit

6 times 6 equals 36

Repeating digit 6

Round to the nearest hundredth $5 \frac{1}{6} = 5.\overline{16} \approx 5.17$

A fraction with a relatively small denominator can result in an equivalent decimal with a repeating digit pattern that is very time consuming to calculate by hand. For instance, the fraction $5/17$ has a 16 digit repeating pattern which is too tedious to calculate by hand without the use of technology. Below $5/17$ is listed as a repeating decimal which is then rounded to two decimal places.

$$\frac{5}{17} = 0.\overline{2941176470588235} \approx 0.29$$

For some common fractions it is useful to know their decimal representations without having to perform the long division process. Below the value of a nickel, dime, quarter, and half dollar are written as fractional parts of a dollar and also in corresponding decimal format in terms of cents.

$$\text{half dollar} = \text{a } \mathbf{half} \text{ of a dollar} = \$\frac{1}{2} = \mathbf{50} \text{ cents} = \mathbf{\$0.50}$$

$$\text{quarter} = \text{a } \mathbf{fourth} \text{ of a dollar} = \$\frac{1}{4} = \mathbf{25} \text{ cents} = \mathbf{\$0.25}$$

$$\text{dime} = \text{a } \mathbf{tenth} \text{ of a dollar} = \$\frac{1}{10} = \mathbf{10} \text{ cents} = \mathbf{\$0.10}$$

$$\text{nickel} = \text{a } \mathbf{twentieth} \text{ of a dollar} = \$\frac{1}{20} = \mathbf{5} \text{ cents} = \mathbf{\$0.05}$$

Below is a list of fractions and their decimal representations that are rounded to two decimal places. If possible, try to memorize these relationships which will come in handy when working with percentages.

$\frac{1}{2} = 0.50$	$\frac{1}{3} = 0.\bar{3} \approx 0.33$	$\frac{2}{3} = 0.\bar{6} \approx 0.67$
$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$	$\frac{1}{5} = 0.20$
$\frac{2}{5} = 0.40$	$\frac{3}{5} = 0.60$	$\frac{4}{5} = 0.80$
$\frac{1}{10} = 0.10$	$\frac{1}{20} = 0.05$	$\frac{1}{100} = 0.01$

Example 7 List from smallest to largest 0.76, 0.7, $\frac{3}{4}$, 0.81 and $\frac{4}{5}$

First write the fractions $\frac{3}{4}$ and $\frac{4}{5}$ as decimals. Create like decimals by adding ending zeros when needed so that all the decimals have the same ending decimal place, in this case the hundredths place. Then list their numerators from smallest to largest 70, 75, 76, 80 and 81. As shown below 0.70, $\frac{3}{4}$, 0.76, $\frac{4}{5}$ and 0.81 are listed from smallest to largest.

$$\begin{array}{rcl}
 0.76 & \xrightarrow{\quad} & 0.70 \\
 0.70 & \xrightarrow{\quad} & 0.75 = \frac{3}{4} \\
 \frac{3}{4} = 0.75 & \xrightarrow{\quad} & 0.76 \\
 0.81 & \xrightarrow{\quad} & 0.80 = \frac{4}{5} \\
 \frac{4}{5} = 0.80 & \xrightarrow{\quad} & 0.81
 \end{array}$$

Exercises 4.3

1-6 Use long division to write the following fractions in decimal format.

1. $\frac{7}{20}$

2. $\frac{3}{8}$

3. $\frac{3}{16}$

4. $8\frac{11}{25}$

5. $7\frac{23}{40}$

6. $3\frac{1}{8}$

For #7-15 round all decimals to two decimal places

7-15 Use long division to write the following fractions in decimal format.

7. $\frac{5}{12}$

8. $\frac{4}{7}$

9. $\frac{8}{9}$

10. $\frac{7}{11}$

11. $\frac{9}{14}$

12. $\frac{2}{13}$

13. $3\frac{7}{16}$

14. $9\frac{1}{12}$

15. $6\frac{3}{11}$

16-24 Write the following fractions in decimal format (without showing steps)

16. $\frac{3}{4}$

17. $\frac{2}{3}$

18. $\frac{4}{5}$

19. $\frac{1}{3}$

20. $\frac{1}{2}$

21. $\frac{3}{5}$

22. $\frac{2}{5}$

23. $\frac{1}{4}$

24. $\frac{1}{20}$

25-30 List the following from smallest to largest

25. 0.53, 0.519, $\frac{1}{2}$ and 0.56

26. 0.33, $\frac{1}{3}$, 0.34 and 0.335

27. $\frac{9}{20}$, 0.4, 0.39 and 0.448

28. 0.2, 0.148, 0.17 and $\frac{3}{20}$

29. $\frac{4}{5}$, 0.79, .801 and $\frac{9}{10}$

30. 0.23, $\frac{1}{4}$, 0.27 and $\frac{6}{25}$