

Section 6.1 Rates

In the previous chapter, ratios are created which compare two quantities measured using the same units by describing how the magnitude (the size) of first quantity relates to the second quantity. In this section rates are created to compare two quantities which are measured using different units. Rates are written as fractions with the indicated units included in the numerator and denominator. To reduce a rate, write the rate in fraction form and reduce the numerator and denominator by cancelling out all the common factors. Since the units of a rate are different, these units cannot be canceled.

Definition A **rate** compares two quantities measured using different units. A rate is written in fraction notation with the units included in the numerator and denominator.

Example 1 Write the following rates as fractions and reduce if possible.

A part time job pays \$200 for 16 hours of work

This rate involves an amount measured in dollars and a time measured in hours. Reduce by the common factor 8. The reduced pay rate indicates that this job pays \$25 every 2 hours.

$$\frac{\$200}{16 \text{ hr}} = \frac{\overset{25}{\cancel{\$200}}}{\underset{2}{\cancel{16} \text{ hr}}} = \frac{\$25}{2 \text{ hr}} \text{ or } \$25 \text{ every 2 hours}$$

500 milligrams of calcium in 3 tablets

This rate involves a mass measured in milligrams and an amount in tablets. This rate is already in reduced form.

$$\frac{500 \text{ mg}}{3 \text{ tab}} \text{ or } 500 \text{ milligrams in 3 tablets}$$

A car driven 302 miles in 6 hours

This rate involves a distance measured in miles and a time measured in hours. Reduce by the common factor 2. The reduced rate indicates that the car is driven at an average speed of 151 miles in 3 hours.

$$\frac{302 \text{ mi}}{6 \text{ hr}} = \frac{\overset{151}{\cancel{302}} \text{ mi}}{\underset{3}{\cancel{6} \text{ hr}}} = \frac{151 \text{ mi}}{3 \text{ hr}} \text{ or } 151 \text{ miles in 3 hours}$$

Definition A **unit rate** compares the first measured quantity to one unit of the second measured quantity. A unit rate is written in fraction format with a denominator of one and in words as the first measured quantity “per” one unit of the second measured quantity.

The following unit rates are written in both fraction format and in words with “per” acting as the separator (fraction bar). Note, when unit rates are written in words the quantity 1 is not usually written in front of the second unit. Unit rates that involve the cost per item such as \$1.47 per can or \$0.63 per ounce are referred to as unit cost.

35 miles per gallon	80 milligrams per tablet	\$12.75 per hour	\$1.47 per can
$\frac{35 \text{ mi}}{1 \text{ gal}}$	$\frac{80 \text{ mg}}{1 \text{ tab}}$	$\frac{\$12.75}{1 \text{ hr}}$	$\frac{\$1.47}{1 \text{ can}}$

Example 2 Write the following rates as unit rates.

An employee earns \$360 for 20 hours of work

This wage rate involves a monetary amount measured in dollars and time measured in hours. Reducing by the common factor 20, the resulting unit rate is \$18 per hour.

$$\frac{\$360}{20 \text{ hr}} = \frac{\overset{18}{\cancel{\$360}}}{\underset{1}{\cancel{20} \text{ hr}}} = \frac{\$18}{1 \text{ hr}} \text{ or } \$18 \text{ per hour}$$

A car driven 160 miles uses 5 gallons of gas

This gas mileage rate involves a length measured in miles and a volume measured in gallons. Reducing by the common factor 5, the resulting unit rate is 32 miles per gallon.

$$\frac{160 \text{ mi}}{5 \text{ gal}} = \frac{\overset{32}{\cancel{160}} \text{ mi}}{\underset{1}{\cancel{5} \text{ gal}}} = \frac{32 \text{ mi}}{1 \text{ gal}} \text{ or } 32 \text{ miles per gallon}$$

80 milligrams of vitamin C in 4 tablets

This rate involves a mass measured in milligrams and an amount measured in tablets. Reducing by the common factor 4, the resulting unit rate is 20 milligrams of vitamin C per tablet.

$$\frac{80 \text{ mg}}{4 \text{ tab}} = \frac{\overset{20}{\cancel{80}} \text{ mg}}{\underset{1}{\cancel{4} \text{ tab}}} = \frac{20 \text{ mg}}{1 \text{ tab}} \text{ or } 20 \text{ milligrams per tablet}$$

In the previous problem all the rates when reduced become unit rates. This does not always occur. To write a rate as a unit rate, first try to reduce as much as possible but if the resulting denominator is not one, then use long division to write the resulting unit rate in decimal form.

Example 3 Write the following rates as unit rates.

A part time job pays \$200 for 16 hours of work

This hourly pay rate involves an amount measured in dollars and a time measured in hours. First reduce by the common factor 8 then divide using long division. As shown below the resulting unit rate is \$12.50 per hour.

$$\frac{\$200}{16 \text{ hr}} = \frac{\overset{25}{\cancel{\$200}}}{\underset{2}{\cancel{16} \text{ hr}}} = \frac{\$12.50}{1 \text{ hr}} \text{ or } \$12.50 \text{ per hour}$$

A car driven 307 miles uses 9 gallons of gas

This gas mileage rate involves a length measured in miles and a volume measured in gallons. Since the rate is in reduced form, using long division as shown below the resulting unit rate is approximately 34 miles per gallon.

$$\frac{307 \text{ mi}}{9 \text{ gal}} \approx \frac{34 \text{ mi}}{1 \text{ gal}} \text{ or } 34 \text{ miles per gallon}$$

500 milligrams of calcium in 3 tablets

This rate involves a mass measured in milligrams and an amount in tablets. Since the rate is in reduced form, using long division as shown below the resulting unit rate is approximately 167 milligrams of calcium per tablet.

$$\frac{500 \text{ mg}}{3 \text{ tab}} \approx \frac{167 \text{ mg}}{1 \text{ tab}} \text{ or } 167 \text{ mg per tablet}$$

$$\begin{array}{r} 34.1 \\ 9 \overline{) 307.0} \\ \underline{-27} \\ 37 \\ \underline{-36} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

$$\begin{array}{r} 166.6 \\ 3 \overline{) 500.0} \\ \underline{-3} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 6 \end{array}$$

Example 4 Sharon purchase 6 chairs for \$460. Find the unit rate (unit cost) which gives the cost per chair rounded to the nearest dollar.

This rate involves a price measured in dollars and amount in chairs. First reduce by the common factor 2 then divide using long division. As shown below the resulting unit rate which gives the cost per unit rounded to the nearest dollar is \$77 per chair.

$$\text{\$460 for 6 chairs} \quad \frac{\$460}{6 \text{ chair}} = \frac{\overset{230}{\cancel{\$460}}}{\underset{3}{\cancel{6} \text{ chair}}} \approx \frac{\$77}{1 \text{ chair}} \text{ or } \$77 \text{ per chair}$$

$$\begin{array}{r} 76.6 \\ 3 \overline{) 230.0} \\ \underline{-21} \downarrow \\ 20 \downarrow \\ \underline{-18} \downarrow \\ 20 \downarrow \\ \underline{-18} \\ 2 \end{array}$$

Example 5 A styling shampoo is sold in two sizes, a small 8 fluid ounce size which sells for \$4.80 and a large 20 fluid ounces size which sell for \$10.40. Find the unit rate (unit cost) which gives the cost per fluid ounce for each size and determine which is priced lower.

These rates involve a price measured in dollars and a volume measured in ounces. Use long division as shown below to find the corresponding cost per ounces. Since \$0.52 per ounce is lower than \$0.60 per ounce, the large size shampoo priced at \$10.40 for 20 ounces is the better deal.

\$4.80 for an 8 ounce size

$$\frac{\$4.80}{8 \text{ oz}} = \$0.60 \text{ per ounce}$$

$$\begin{array}{r} .60 \\ 8 \overline{) 4.80} \\ \underline{-48} \downarrow \\ 00 \\ \underline{-00} \\ 0 \end{array}$$

\$10.40 for a 20 ounce size

$$\frac{\$10.40}{20 \text{ oz}} = \$0.52 \text{ per ounce}$$

$$\begin{array}{r} .52 \\ 20 \overline{) 10.40} \\ \underline{-100} \downarrow \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Exercises 6.1

1-10 Write the following rates as fractions in **reduced form**.

1. A job pays \$230 for 20 hours worked.
2. \$12 is the cost of a case of 8 cans of beans.
3. A car travels 400 miles using 12 gallons of gas.
4. A plane travels 1850 miles in 4 hours.
5. A car travels 320 miles in 6 hours.
6. 90 calories are expended during a 25-minute walking workout.
7. An infusion pump delivers 400 milliliters of a saline solution in 6 hours.
8. An IV delivers 250 drops of fluid in 20 minutes.
9. The 2012 birth rate in the U.S. is approximately 62 births for every 1000 women of childbearing age.
10. The mortality rate in 2012 in the U.S. is approximately 800 deaths for every 100,000 people.

11-12 Identify which of the following rates are written as unit rates.

11. $\frac{\$12}{1 \text{ ticket}}$, \$3.20 for 8 ounces , $\frac{350 \text{ mi}}{5.5 \text{ hr}}$, 2.6 people per household
12. $\frac{\$25}{12 \text{ cans}}$, 32 miles per gallon , $\frac{80 \text{ beats}}{1 \text{ min}}$, 25 milligrams per tablet

13-18 Write the following rates as **units rates**.

13. 48 grams of fat are in 4 servings of lasagna.
14. \$300 earned for working 20 hours.
15. A car travels 400 miles using 10 gallons of gas.
16. 200 milligrams of a medication are contained in 4 tablets.
17. 200 calories are expended during a 40-minute walking workout.
18. Al picks 600 oranges from 3 trees in his backyard.

19-26 Write the following rates as **units rates**. Round final answers to the nearest whole number unless otherwise indicated.

19. A job pays \$230 for 20 hours worked. Round to the nearest cent.
20. \$12 is the cost of a case of 8 cans of beans. Round to the nearest cent.
21. A car travels 400 miles using 12 gallons of gas.
22. A plane travels 1850 miles in 4 hours.
23. A car travels 320 miles in 6 hours.
24. 90 calories are expended during a 25-minute walking workout.
25. An infusion pump delivers 400 milliliters of a saline solution in 6 hours.
26. An IV delivers 250 drops of fluid in 20 minutes.

27-30 Find the **unit cost** (the cost per item) rounded to the nearest cent.

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|---------------------------------------|--------------------------------------|
| 27. \$3.60 for 12 ounces of shampoo | 28. \$6.40 for 8 bars of soap |
| 29. \$31.85 for 8 gallons of gasoline | 30. \$2.23 for a 5 minute phone call |

31-34 Determine which of the following is the better deal. Show steps.

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|---------------------------------------|----|-----------------------------------|
| 31. \$4.20 for 10 ounces of shampoo | OR | \$9.36 for 24 ounces of shampoo |
| 32. \$4.40 for 4 bars of soap | OR | \$2.16 for 2 bars of soap |
| 33. \$3.75 for 6 sodas | OR | \$15.36 for 24 sodas |
| 34. \$1.75 for a 40 minute phone call | OR | \$2.52 for a 60 minute phone call |