Section 6.3   U.S. customary units

The metric system is part of the international system of units (SI system) that defines units to measure lengths, volumes, masses, times, amounts, electric currents, and luminous intensities. The ease of use of the metric system has made it the standard unit of measurement worldwide, with the glaring exception of the United States which still uses the U.S. customary system (based on the English system) for many everyday calculations. But even in the United States, the metric system is standard for scientific calculations in biological, chemical, and medical applications. This section will review the U.S. customary system. Below the conversion factors for length, volume capacity and mass are listed.

Length	Volume capacity	Mass
12 inches = 1 foot	3 teaspoons = 1 tablespoon	16 ounces = 1 pound
3 feet = 1 yard	2 tablespoons = 1 fluid ounce	2000 pounds = 1 ton
1760 yards = 1 mile	8 fluid ounces = 1 cup	
	2 cups = 1 pint	
	2 pints = 1 quart	
	4 quarts = 1 gallon	

In this textbook conversions between units are performed using the dimensional analysis technique. To convert using the dimensional analysis technique the calculation is written as a product of fractions with the conversion factor rate written as a fraction with the output unit (converted to unit) in the numerator and input unit (converted from unit) in the denominator. When the multiplication calculation shown below is performed the converted from units cancel out leaving the appropriate converted to unit as the final answer. The follow phrase describes this process, "***write the conversion factor rate with the converted to unit on the top***".

$$\underbrace{\left(\frac{\text{converted to unit}}{\cancel{\text{converted from unit}}} \right)}_{conversion\ factor} \times \frac{\left(\cancel{\text{converted from unit}} \right)}{1}$$

Example 1   Convert 6 yards into feet

The convert to unit is feet.

Conversion factor written with the converted to unit feet on the top $\dfrac{3 \text{ ft}}{1 \text{ yd}}$

This conversion factor multiplied times the 6 yards results with the yard unit being canceled as shown below. 6 yards is equal to 18 feet.

$$\frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{6 \text{ yd}}{1} \;=\; 18 \text{ feet}$$

Example 2   Convert 30 inches into feet

The convert to unit is feet.

Conversion factor written with the converted to unit feet on the top $\dfrac{1 \text{ ft}}{12 \text{ in}}$

This conversion factor multiplied times the 30 inches results with the inch unit being canceled as shown below. To multiply the fractions first cancel the greatest common factor 6 from the 30 and 12 then write the resulting fraction 5/2 in decimal form. 30 inches equal to 2.5 feet.

$$\frac{1 \text{ ft}}{12 \text{ in}} \times \frac{30 \text{ in}}{1} \;=\; \frac{1 \text{ ft}}{\overset{}{\underset{2}{12}} \text{ in}} \times \frac{\overset{5}{30} \text{ in}}{1} \;=\; \frac{5}{2} \text{ ft} \;=\; 2.5 \text{ feet}$$

Example 3   Convert 10 quarts into gallons

The convert to unit is gallon.

Conversion factor written with the converted to unit gallon on the top $\dfrac{1 \text{ gal}}{4 \text{ qt}}$

This conversion rate multiplied times the 10 quarts results with the quart unit being canceled as shown below. To multiply the fractions first cancel the greatest common factor 2 from the 10 and 4 then write the resulting fraction 5/2 in decimal form. 10 quarts is equal to 2.5 gallons.

$$\frac{1 \text{ gal}}{4 \text{ qt}} \times \frac{10 \text{ qt}}{1} \;=\; \frac{1 \text{ gal}}{\overset{}{\underset{2}{4}} \text{ qt}} \times \frac{\overset{5}{10} \text{ qt}}{1} \;=\; \frac{5}{2} \text{ gal} \;=\; 2.5 \text{ gal}$$

Example 4   A bottle of olive oil contains 34 fluid ounces. Find how many cups
              can be filled with oil from this bottle.

The convert to unit is cup.

Conversion factor written with the converted to unit cup on the top $\dfrac{1\text{ c}}{8\text{ fl oz}}$

This conversion factor multiplied times the 34 fluid ounces results with the fluid
ounce unit being canceled as shown below. To multiply the fractions first cancel the
greatest common factor 2 from the 34 and 8 then write the resulting fraction 17/4 as
a mixed number or in decimal form. 34 fluid ounces is equal to 4 1/4 or 4.25 cups.

$$\frac{1\text{ c}}{8\text{ fl oz}} \times \frac{34\text{ fl oz}}{1} \;=\; \frac{1\text{ c}}{\cancel{8}\,_4\;\cancel{\text{fl oz}}} \times \frac{\overset{17}{\cancel{34}}\;\cancel{\text{fl oz}}}{1} \;=\; \frac{17}{4}\text{ c} \;=\; 4\frac{1}{4}\text{ c}\quad\text{or}\quad 4.25\text{ c}$$

Mass is the technical physics name for the concept that is referred to as weight in
everyday language. Technically the weight of an object is a measurement that
dependents on gravity which is based on the distance the object is located from the
center of the earth while the mass of an object remains the same at any elevation. On
the surface of the earth the numerical value of the mass and weight are basically the
same. For a person whose mass is 120 pounds on earth their mass would remain at
120 pounds on the moon while their weight would only be only approximately 20
pounds on the moon.   In this chapter, the correct wording of mass is used in
applications except for those involving the mass of humans where the everyday
language of weight is used.

Example 5   Convert 24 ounces into pounds

The convert to unit is pound.

Conversion factor written with the converted to unit pound on the top $\dfrac{1\text{ lb}}{16\text{ oz}}$

This conversion factor multiplied times the 24 ounces results with the ounce unit
being canceled as shown below. To multiply the fractions first cancel the greatest
common factor 8 from the 24 and 16 then write the resulting fraction 3/2 in decimal
form. 24 ounces is equal to 1.5 pounds.

$$\frac{1\text{ lb}}{16\text{ oz}} \times \frac{24\text{ oz}}{1} \;=\; \frac{1\text{ lb}}{\cancel{16}\,_2\;\cancel{\text{oz}}} \times \frac{\overset{3}{\cancel{24}}\;\cancel{\text{oz}}}{1} \;=\; 1.5\text{ lb}$$

Example 6   An African elephant can weigh as much as 6.6 tons. Convert this mass into pounds.

The convert to unit is pound.

Conversion factor written with the converted to unit pound on the top $\dfrac{2000 \text{ lb}}{1 \text{ ton}}$

This conversion factor multiplied times the 6.6 tons results with the ton unit being canceled as shown below. To evaluate this expression multiply 2000 times 6.6, which results with an African elephant having a mass of 13,200 pounds.

$$\frac{2000 \text{ lb}}{1 \text{ ton}} \times \frac{6.6 \text{ ton}}{1} \;=\; 13{,}200 \text{ lb}$$

For some conversions a direct conversion factor between the converted to unit and the converted from unit is not listed in a given reference table, but other conversion factors are listed which serve as "middleman" between these units. In these cases multiple conversion factors are employed and arranged in such a way that all units besides the output unit cancel out.

Example 7   A trail that connects the North Rim of the Grand Canyon to the Colorado River descends 5850 feet in elevation. Convert this descend in elevation into miles.

A direct conversion factor between feet and miles is not listed in this section, but both of these units are defined in terms of yards. Thus, the yard unit will serve as the middleman connecting the feet and miles unit. First convert feet into the yards then convert the yard into miles. Write the conversion factors 3 feet equals 1 yard and 1760 yards equal 1 mile in fraction form to create the expression shown below with the convert to unit mile in the numerator. To evaluate use a calculator to find the product of the denominators 1760 and 3 which equals 5280 then divide 5850 by 5280. This trail descends 5850 feet is approximately 1.11 miles.

$$\frac{1 \text{ mi}}{1760 \text{ yd}} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{5850 \text{ ft}}{1} \;=\; \frac{5850}{5280} \text{ mi} \;\approx\; 1.11 \text{ mi}$$

Example 8   Convert 4 fluid ounces into teaspoons

A direct conversion factor between fluid ounces and teaspoons is not listed in this section, but these units are defined in terms of tablespoons. Thus, the tablespoon unit will serve as the middleman connecting the fluid ounce and teaspoon unit. First convert fluid ounces into the tablespoons then convert the tablespoons into teaspoons. Write the conversion factors 1 fluid ounce equals 2 tablespoons and 1 tablespoon equal 3 teaspoons in fraction form to create the expression shown below with the convert to unit teaspoon in the numerator. To evaluate this expression multiply the numerators 3, 2, and 4. 4 fluid ounces is equal to 24 teaspoons.

$$\frac{3\ \text{tsp}}{1\ \text{Tbs}} \times \frac{2\ \text{Tbs}}{1\ \text{floz}} \times \frac{4\ \text{floz}}{1} = 24\ \text{tsp}$$

In the U.S. human heights are commonly written with both the feet and inches units. For instances, a person height is written as 5 feet 10 inches. When listing heights using feet and inches, the inches represent the remainder when the total inches are divided by the conversion factor of 12 inches equal 1 foot. For converting between inches and feet and total inches measurements the formal dimensional analysis steps will not be written. To convert 77 inches into feet and inches divide by 12 and write the answer in mixed number format. As shown below, 77 inches equals 6 5/12 feet which is written as 6 feet 5 inches. The remainder in this case 5 is written as the inches part of the feet and inches measurement.

$$
\begin{array}{r}
6\ \text{R}\mathbf{5} \\
12\overline{\smash{\big)}\ \mathbf{7\,7}} \\
-\underline{7\,2} \\
\mathbf{5}
\end{array}
$$

Example 9   Jane is 68 inches tall. Find her height in feet and inches.

Divide 68 by 12 as shown below results with 5 R8 or 5 8/12 feet. Jane is 5 feet 8 inches tall.

$$
\begin{array}{r}
5\ \text{R}\mathbf{8} \\
12\overline{\smash{\big)}\ \mathbf{6\,8}} \\
-\underline{6\,0} \\
\mathbf{8}
\end{array}
$$

Example 10   Sal is 5 feet 11 inches tall. Find his height in inches.

To convert from feet and inches into total inches multiply the feet by 12 and add the inches. As shown below, Sal is 71 inches tall.

$$5(12) + 11 = 60 + 11 = 71$$

Exercises 6.3

Length	Volume capacity	Mass
12 inches = 1 foot	3 teaspoons = 1 tablespoon	16 ounces = 1 pound
3 feet = 1 yard	2 tablespoons = 1 fluid ounce	2000 pounds = 1 ton
1760 yards = 1 mile	8 fluid ounces = 1 cup	
	2 cups = 1 pint	
	2 pints = 1 quart	
	4 quarts = 1 gallon	

1-6   Use dimensional analysis to convert between U.S. customary length units using the appropriate conversion factor listed above.   (*calc* indicates use calculator)

1.	6 feet into inches	2.	66 inches into feet
3.	5 yards into feet	4.	24 feet into yards
5.	3 miles to yards	6.	10,000 yards into miles (*calc*)

7-12   Use dimensional analysis to convert between U.S. customary volume units using the appropriate conversion factor listed above.

7.	20 fluid ounces into cups	8.	3 fluid ounces into tablespoons
9.	2.5 gallons into quarts	10.	12 teaspoons into tablespoons
11.	3 cups into fluid ounces	12.	3 quarts into pints

13-18 Use dimensional analysis to convert between U.S. customary mass units using the appropriate equivalency listed above.

13.	24 ounces into pounds	14.	3 pounds into ounces
15.	24 ounces into pounds	16.	12 ounces into pounds
17.	4.5 tons into pounds	18.	5000 pounds into tons

19-24 Use dimensional analysis to convert between U.S. customary units. Since a direct conversion factor is not listed **use multiple conversion factors.**

19.	6 yards into inches	20.	10,000 feet into miles (*calc*)
21.	3 quarts into cups	22.	3 gallons into pints
23.	3 cups into tablespoons	24.	18 teaspoons into fluid ounces

25-28 Write the following human heights in feet and inches.

25.   58 inches   26.   62 inches

27.   68 inches   28.   73 inches

29-32 Write the following human heights in inches.

29.   4 feet 8 inches   30.   5 feet 3 inches

31.   5 feet 11 inches   32.   6 feet 5 inches